For Reference

NOT TO BE TAKEN FROM THIS ROOM

Ex libris
universitatis
albertaensis

QUAECUMQUE VERA

THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR .... MICHAEL ABIMBOLA LAWAL ....

TITLE OF THESIS .... Design of Discrete-Time Water Quality ....

.... Controllers for a Polluted River ....

.... System ....

DEGREE FOR WHICH THESIS WAS PRESENTED .... MASTER OF SCIENCE ....

YEAR THIS DEGREE GRANTED .... 1977 ....

Permission is hereby granted to THE UNIVERSITY OF ALBERTA LIBRARY to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly or scientific research purposes only.

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission.

THE UNIVERSITY OF ALBERTA

DESIGN OF DISCRETE-TIME WATER QUALITY CONTROLLERS FOR A POLLUTED RIVER SYSTEM

by

© MICHAEL A. LAWAL

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

EDMONTON, ALBERTA

FALL, 1977

Digitized by the Internet Archive
in 2019 with funding from
University of Alberta Libraries

https://archive.org/details/Lawal1977

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled DESIGN OF DISCRETE-TIME WATER QUALITY CONTROLLERS FOR A POLLUTED RIVER SYSTEM submitted by MICHAEL A. LAWAL in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE IN ELECTRICAL ENGINEERING.

ABSTRACT

Procedures for designing feedback controllers for maintaining a prescribed level of dissolved oxygen in a single reach of a polluted river system are described in this thesis. A single control scheme in which artificial instream aeration is used as the control and a two-control scheme using aeration and effluent discharge as the two controls are developed. A discrete-time model of the polluted river system used in the controller design is derived first starting from an available continuous time model.

Unlike continuous-time controllers developed by others previously, the advantage of the discrete-time controllers is that the output DO (dissolved oxygen) does not have to be monitored at all times. This is very attractive from a practical point of view.

The effect of seasonal variations in the temperature of the water on the performance of the digital controllers developed in this thesis as well as the continuous controllers reported in the literature is also investigated by means of computer simulations.

ACKNOWLEDGEMENTS

The author wishes to express his thanks to Dr. V. Gourishankar for his guidance during the preparation of this work. The author expresses his thanks to Dr. P. Kudva for his valuable help and suggestions regarding the computer simulations presented in this thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure		Page

CHAPTER 1

INTRODUCTION

1.1 The Environment

Man has not paid much attention to his environment until recently. In as much as growth, achieved by increased industrialization and urbanization, is desired in our societies, it is essential that steps are taken to alleviate the problems that accompany such growth. One of these problems is environmental pollution. The effect of man's activities on environment in or near industrial and urban centers is just being known. The increase in the rate of discharge of industrial and urban waste into the environment has not been matched by an equivalent increase in the assimilative capacity of the environment, with the consequent degradation of the basic requirements for life itself; (viz) air and water.

1.2 Sources of Environmental Pollution

It is evident that the soil which is the main source of man's food has been turned into a dumping ground for pollutants such as solid refuse from both urban and rural sources, chemical agents used in the production of food and other industrial processes and particles settling out of the air as a result of air pollution. It is now

known that some of these pollutants cause harmful residues to be concentrated in the food chain [9][1].

The pollution of the air is caused by the burning of fuels for transportation, heat and power and the emission of sulphur dioxide, particle matter, carbon monoxide, photo-chemical oxidants, hydrocarbons and nitrogen dioxide into the air by industrial processes that give off these gases as waste products. Environmentalists speculate that the oxygen-carbon dioxide balance might be upset due to these pollutants [9]. In the case of water pollution, the use of lakes, rivers and the oceans by municipalities and industries for the dilution of their liquid wastes and as dumping ground for their solid wastes has made these places unsafe for recreational activities such as swimming and most important as a source of food; fish.

1.3 Water Pollution

There are many factors to be considered in assessing the quality of water in a river or stream. These are:

Physical factor: The color of the water.

Chemical factors: Amount of the various chemical compounds present, e.g. nitrates, chlorides, phosphates, etc.

Biological factors: The composition of life in the river. It is a fact that in a normally healthy river, there is a

---

[1] The number in square brackets refer to the references listed under bibliography at the end of this thesis.

biodynamic cycle which results in a balance between the plant and animal life. Ruth Patrick [13] has developed a system for assessing the degree of pollution of a river from an analysis of the various groups of organisms present in the river.

<u>Biochemical factors</u>: The biochemical oxygen demand, BOD and the dissolved oxygen, DO. The biochemical oxygen demand is defined as the amount of oxygen required by bacteria to decompose a given amount of organic waste under aerobic* conditions at a given temperature. Dissolved oxygen, DO, is defined as the amount of oxygen absorbed by a river at a given temperature.

Of all the factors stated above BOD and DO are generally accepted as the primary indicators of water quality. The reasons for this will be clear from a discussion of the pollution of a river by organic wastes as outlined by Brinley [13].

Organic waste dumped into the river is actually broken down by bacteria present in the river. This decomposition is characterized by three phases. These are:

(i) <u>Active phase</u>

In this phase there is increased bacterial activity resulting in increased respiration and therefore increased oxygen consumption. The biochemical oxygen

---

* Reaction occurring only in the presence of oxygen.

demand is high and thus a low level of dissolved oxygen results. The biodynamic cycle is disrupted and organisms that require a lot of oxygen in order to survive become extinct.

(ii) Intermediate phase

Since most of the waste has already been decomposed in the active phase, the amount of energy required by the bacteria to break down the remaining waste is greatly reduced in this phase. Dissolved oxygen level rises as a result of this and some living organisms start to return to the river.

(iii) Recovery phase

As the waste is progressively decomposed, the water in the river becomes clearer, and the biodynamic cycle comes into effect once again.

Thus it can be seen that low levels of biochemical oxygen demand and high levels of dissolved oxygen are characteristics of a river free from organic waste pollution.

1.4 Application of Automatic Control Theory to Water Pollution

It is evident from the last section that it takes some time for the river to recover from the effects of organic waste pollution. Thus the greater the rate of waste discharged into the river, the less the river system is able to assimilate it . In the past few years some researchers have focused their attention on the engineering

aspects of improving and controlling the water quality in rivers which are subjected to organic waste dumping. Some of them [8, 22] have formulated the water pollution problem as an optimization problem. Not only is such a formulation complicated but in most cases the solutions arrived at are not easy to implement. Young and Beck [24] have used a simpler approach to the problem. They have used the pole assignment technique to design a controller that will maintain a prescribed level of DO in a single reach of a river system using complete state feedback. Ramar and Gourishankar [18,19] have gone further and designed controllers using output feedback (DO) only for pole assignment.

They have also used the minimization of a sensitivity function to obtain a better response than those obtained by Young and Beck. Kudva and Gourishankar [11] in their work in this field have designed an observer to estimate the output BOD whose exact and continuous measurement is difficult and have been able to show that an increased effluent discharge on the whole over a period of time can be achieved by their method.

In all these cases the section of the river whose water quality is to be maintained has been modelled as a continuous system. This means that the output DO has to be monitored or the BOD estimated all the time. A more practical approach would be to monitor the output DO only at certain predetermined instants of time and use this

information to control the output DO during the period between samplings. Such a discrete time controller would be easier to implement, particularly in this age of digital computers.

1.5 Scope of this Thesis

The work reported in this thesis is primarily concerned with the design of a direct digital controller (DDC) to maintain a prescribed level of DO in a single reach of a river system subjected to dumping of organic waste.

A comparative study is also made between the performance of the digital controller and that of the continuous time controllers designed by Ramar and Gourishankar, or Kudva and Gourishankar [18, 19, 10].

In Chapter Two of this thesis, a discrete-time state space model for a single reach of a river system is derived. Two cases are treated. One with artificial aeration as the only control and the other with aeration plus effluent discharge as the two controls.

Chapter Three describes the design of the digital controllers for the two cases mentioned above, using the pole-assignment and output (DO) feedback.

The effect of seasonal temperature variation on the performance of the continuous and digital controllers in maintaining water quality is investigated in Chapter Four.

In Chapter Five, the results are summarized, comparisons are made and conclusions drawn. Suggestions for further research are also given.

CHAPTER 2

DISCRETE-TIME STATE SPACE MODEL OF A RIVER SYSTEM

2.1 Continuous-Time Model

As mentioned in the previous chapter the most commonly accepted parameters for assessing water quality are the concentration of dissolved oxygen (DO) and the biochemical oxygen demand (BOD) in a section of the river. A number of different models have been derived over the past few years starting from the general diffusion equation and using BOD and DO as variables [1, 4, 6, 21, 22].

One such model used by many researchers is given in equation (2-1) below.

$$\text{DO:} \quad \frac{dx_1(t)}{dt} = -\left(a_1 + \frac{Q + Q_E}{V_m}\right)x_1(t) - a_2x_2(t) + \left(\frac{Q}{V_m}\right)C_i(t) + a_1C_s - D_B + \frac{Q_EC_E}{V_m}$$

$$\text{BOD:} \quad \frac{dx_2(t)}{d(t)} = -\left(a_2 + \frac{Q + Q_E}{V_m}\right)x_2(t) + \left(\frac{Q}{V_m}\right)L_2(t) + L_A + \left(\frac{Q_E}{V_m}\right)L_E \qquad (2\text{-}1)$$

Where

$x_1(t)$ is the output (downstream) DO in mg/ℓ.

$x_2(t)$ is the output BOD in mg/ℓ.

$L_i(t)$ is the input (upstream) BOD in mg/ℓ.

$C_i(t)$ is the input DO in mg/ℓ.

C_s is the saturation value of DO in mg/ℓ.

L_E is the BOD in the effluent in mg/ℓ.

C_E is the DO in the effluent in mg/ℓ.

L_A is the mean rate of addition of BOD by local runoff.

D_B is the net rate of removal of DO by the combined action of photosynthesis and respiration.

V_m is the mean volume of the water held in the reach in m^3.

Q is the volumetric flow rate in m^3/day.

Q_E is the volumetric flow rate of the effluent in m^3/day.

a_1 is the reaeration rate in day^{-1}.

a_2 is the BOD decay rate in day^{-1}.

Equation (2-1) can be rewritten in vector matrix form as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -(a_1 + \frac{Q + Q_E}{V_m}) & -a_2 \\ 0 & -(a_2 + \frac{Q + Q_E}{V_m}) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} +$$

$$\begin{bmatrix} \frac{Q}{V_m} & 0 \\ 0 & \frac{Q}{V_m} \end{bmatrix} \begin{bmatrix} C_i(t) \\ L_i(t) \end{bmatrix} + \begin{bmatrix} a_1 C_s - D_B + \frac{Q_E C_E}{V_m} \\ L_A + \frac{Q_E L_E}{V_m} \end{bmatrix} \quad (2\text{-}2)$$

where $\dot{x}_1(t)$ and $\dot{x}_2(t)$ represent the differentiation of $x_1(t)$ and $x_2(t)$ with respect to time.

The continuous-time model given in equation (2-2) does not include the control term. Introducing this term, the continuous-time model with artificial aeration as the only control becomes

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\left(a_1 + \dfrac{Q + Q_E}{V_m}\right) & -a_2 \\ 0 & -\left(a_2 + \dfrac{Q + Q_E}{V_m}\right) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} +$$

$$\begin{bmatrix} \dfrac{Q}{V_m} & 0 \\ 0 & \dfrac{Q}{V_m} \end{bmatrix} \begin{bmatrix} C_i(t) \\ L_i(t) \end{bmatrix} + \begin{bmatrix} a_1 C_s - D_B + \dfrac{Q_E C_E}{V_m} \\ L_A + \dfrac{Q_E L_E}{V_m} \end{bmatrix} +$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} U_A(t) \qquad (2\text{-}3)$$

Where $U_A(t)$ is the aeration control.

Another method of controlling the output DO in the river is to use not only artificial aeration but also controlled dumping of effluents as the second control. By effluent discharge control is meant the regulation of the amount of waste discharged into the river. A good control parameter would be the deviation of the quantity of waste discharged Q_E from a nominal value $\overline{Q}_E$, i.e.

$$Q_E = \overline{Q}_E + \delta Q_E \qquad (2\text{-}4)$$

Substituting equation (2-4) into the continuous-time model represented by equation (2-1) and adding the aeration control term we have

$$\dot{x}_1(t) = -\left(a_1 + \frac{Q + \bar{Q}_E + \delta Q_E}{V_m}\right)x_1(t) - a_2x_2(t)$$

$$+ \frac{Q}{V_m} C_i(t) + a_1C_s - D_B + \left(\frac{\bar{Q}_E + \delta Q_E}{V_m}\right)C_E + U_A(t)$$

$$\dot{x}_2(t) = -\left(a_2 + \frac{Q + \bar{Q}_E + \delta Q_E}{V_m}\right)x_2(t) + \frac{Q}{V_m} L_i(t)$$

$$+ \left(\frac{\bar{Q}_E + \delta Q_E}{V_m}\right)L_E + L_A \qquad (2\text{-}5)$$

$$\text{Let } U_E = \frac{\delta Q_E}{V_m} \qquad (2\text{-}5a)$$

Substituting equation (2-5a) into (2-5), equation (2-5) can be written in vector matrix form as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\left(a_1 + \dfrac{Q + \bar{Q}_E}{V_m} + U_E\right) & -a_2 \\ 0 & -\left(a_2 + \dfrac{Q + \bar{Q}_E}{V_m} + U_E\right) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} +$$

$$\begin{bmatrix} \dfrac{Q}{V_m} & 0 \\ 0 & \dfrac{Q}{V_m} \end{bmatrix} \begin{bmatrix} C_i(t) \\ L_i(t) \end{bmatrix} + \begin{bmatrix} a_1C_s - D_B + \dfrac{\bar{Q}_EC_E}{V_m} \\ L_A + \dfrac{\bar{Q}_EL_E}{V_m} \end{bmatrix} +$$

$$\begin{bmatrix} 1 & C_E \\ 0 & L_E \end{bmatrix} \begin{bmatrix} U_A(t) \\ U_E(t) \end{bmatrix} \qquad (2\text{-}6)$$

where $U_A(t)$ is the aeration control and

$U_E(t)$ is the effluent discharge control.

Equation (2-6) is the continuous-time model for two controls.

2.2 Discrete-Time Models

The river system to be controlled is inherently a continuous process. In this case the digital signal obtained by sampling the error between the actual output and the desired output will first be processed by the digital controller; the output of the digital controller f*(t), which is a pulse train is then smoothed by a data-reconstruction device before the signal is applied to the continuous process, the river system.

Figure 2.1 shows the schematic diagram of this control scheme. The data-reconstruction device is a zero-order hold.

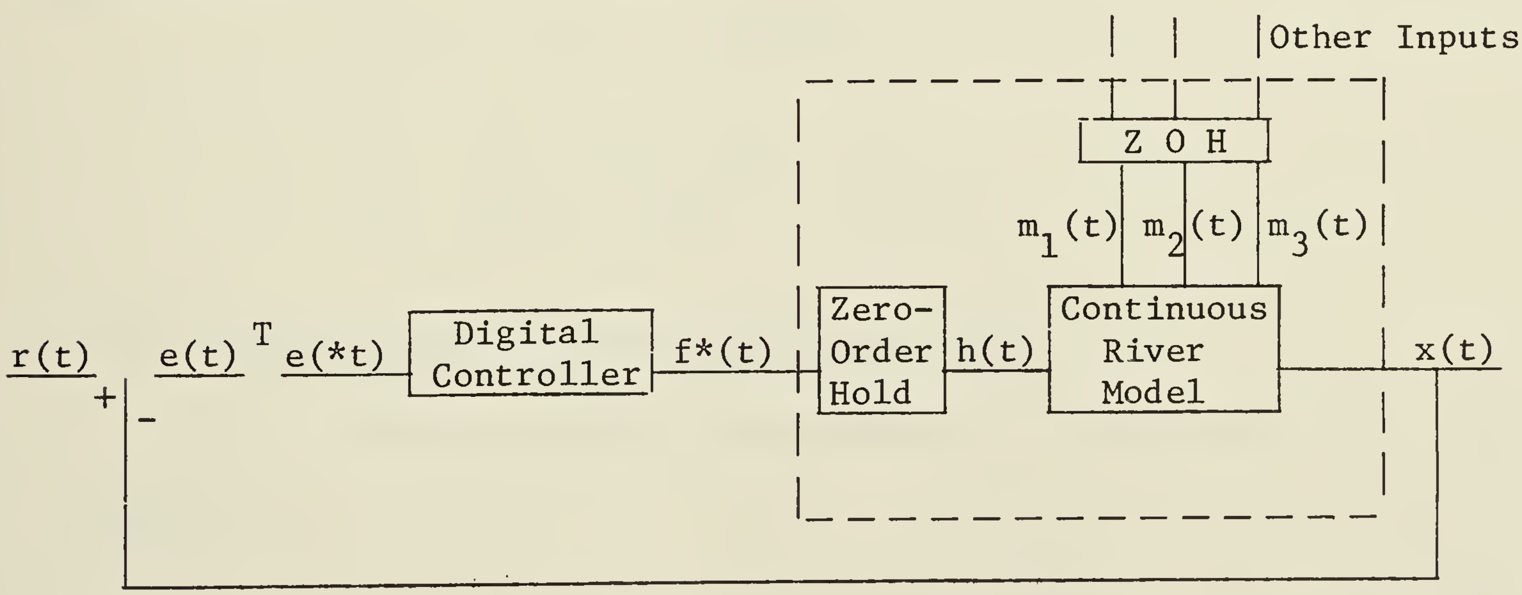


Figure 2.1 Closed Loop Discrete-Time System

In continuous-time systems, the state equations are a set of first-order differential equations. For discrete-

time systems however, the state equations are a set of first-order difference equations. A set of first-order difference equations can be obtained from the set of first-order differential equations by taking a look at the combination of the zero-order hold devices and the continuous system shown in the dotted box in figure 2.1.

Let the other inputs be $C_i(t)$, $L_i(t)$ and θ.

$$h(t) = f(KT) = U_A(KT) \text{ for } KT \leq t \leq (K+1)T \qquad (2\text{-}7)$$

Since h(t) is the input to the continuous-time model.

$$\text{Also} \quad m_1(t) = C_i(KT) \qquad (2\text{-}8)$$

$$m_2(t) = L_i(KT) \qquad (2\text{-}9)$$

$$m_3(t) = \theta(KT) \qquad (2\text{-}10)$$

for $KT \leq t \leq (K+1)T$

where

$$\theta = \begin{bmatrix} a_1 C_s - D_B + \dfrac{Q + Q_E}{V_m} \\ L_A + \dfrac{Q + Q_E}{V_m} \end{bmatrix}$$

2-2a <u>Artificial aeration control only</u>

Taking Laplace transforms of equation (2-3) we have

$$s\,\underline{X}(s) - \underline{x}(t_o) = \begin{bmatrix} -\left(a_1 + \dfrac{Q + Q_E}{V_m}\right) & -a_2 \\ 0 & -\left(a_2 + \dfrac{Q + Q_E}{V_m}\right) \end{bmatrix} \underline{X}(s) +$$

$$\begin{bmatrix} \frac{Q}{V_m} & 0 \\ 0 & \frac{Q}{V_m} \end{bmatrix} \begin{bmatrix} C_i(s) \\ L_i(s) \end{bmatrix} + \frac{\theta}{s} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_A(s) \tag{2-11}$$

where

$$\underline{x}(t_o) = \begin{bmatrix} x_1(t_o) \\ x_2(t_o) \end{bmatrix} \quad \underline{X}(s) = \begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} \quad \text{and}$$

$$\theta = \begin{bmatrix} a_1 C_s - D_B + \frac{Q + Q_E}{V_m} \\ L_A + \frac{Q + Q_E}{V_m} \end{bmatrix}$$

$$\text{Let} \quad A = \begin{bmatrix} -\left(a_1 + \frac{Q + Q_E}{V_m}\right) & -a_2 \\ 0 & -\left(a_2 + \frac{Q + Q_E}{V_m}\right) \end{bmatrix}$$

Solving for $\underline{X}(s)$ we have

$$\underline{X}(s) = (sI - A)^{-1} \left[\underline{x}(t_o) + \begin{bmatrix} \frac{Q}{V_m} & 0 \\ 0 & \frac{Q}{V_m} \end{bmatrix} \begin{bmatrix} C_i(s) \\ L_i(s) \end{bmatrix} + \frac{\theta}{s} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_A(s) \right] \tag{2-12}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{(s + K_1)} & \frac{-a_2}{(s + K_1)(s + K_2)} \\ 0 & \frac{1}{s + K_2} \end{bmatrix} \tag{2-13}$$

where
$$K_1 = a_1 + \frac{Q + Q_E}{V_m}$$

$$K_2 = a_2 + \frac{Q + Q_E}{V_m}$$

Also taking Laplace transforms of equations (2-7), (2-8), (2-9) and (2-10) we have

$$H(s) = \frac{U_A(KT)}{s}$$

$$M_1(s) = \frac{C_i(KT)}{s}$$

$$M_2(s) = \frac{L_i(KT)}{s}$$

$$M_3(s) = \frac{\theta(KT)}{s} \qquad (2\text{-}14)$$

Substituting equations (2-13) and (2-14) into (2-12) and taking inverse Laplace transforms, the resulting equation can be written in vector matrix form as

$$\underline{x}(t) = \Phi(t - t_o)\,\underline{x}(t_o) + B(t - t_o)\,\underline{w}(KT) + D(t - t_o) + E(t - t_o)U_A(KT) \qquad (2\text{-}15)$$

where

$$\Phi(t - t_o) = \begin{bmatrix} e^{-K_1(t - t_o)} & \frac{-a_2}{a_2 - a_1}\left[e^{-K_1(t - t_o)} - e^{-K_2(t-t_0)}\right] \\ 0 & e^{-K_2(t-t_o)} \end{bmatrix} \qquad (2\text{-}16)$$

$$B(t-t_o) = \begin{bmatrix} \dfrac{(\frac{Q}{V_m})[1-e^{-K_1(t-t_o)}]}{K_1} & \dfrac{-a_2(\frac{Q}{V_m})}{K_1K_2}\left[1-\dfrac{K_2e^{-K_1(t-t_o)}}{a_2-a_1} + \dfrac{K_1e^{-K_2(t-t_o)}}{a_2-a_1}\right] \\ 0 & \dfrac{(\frac{Q}{V_m})[1-e^{-K_2(t-t_o)}]}{K_2} \end{bmatrix} \qquad (2\text{-}17)$$

$$D(t-t_o) = \begin{bmatrix} (\frac{\theta_{x1}}{K_1})[1-e^{-K_1(t-t_o)}] - \dfrac{a_2\theta_{x2}}{K_1K_2}\left[1 - \dfrac{K_2e^{-K_1(t-t_o)}}{a_2-a_1} + \dfrac{K_1e^{-K_2(t-t_o)}}{a_2-a_1}\right] \\ \dfrac{\theta_{x2}[1-e^{-K_2(t-t_o)}]}{K_2} \end{bmatrix} \qquad (2\text{-}18)$$

$$E(t-t_o) = \begin{bmatrix} \dfrac{1-e^{-K_1(t-t_o)}}{K_1} \\ 0 \end{bmatrix} \qquad (2\text{-}19)$$

$$\underline{w}(KT) = \begin{bmatrix} C_i(KT) \\ L_i(KT) \end{bmatrix} \qquad (2\text{-}20)$$

$$\theta_{x1} = a_1C_s - D_B + \frac{Q_EC_E}{V_m} \qquad \theta_{x2} = L_A + \frac{Q + Q_E}{V_m}$$

$$K_1 = a_1 + \frac{Q + Q_E}{V_m} \qquad K_2 = a_2 + \frac{Q + Q_E}{V_m}$$

Let $t_o = KT$, $t = (K+1)T$, therefore $t - t_o = T$ which is the sampling interval. Substituting for t, t_o and $t - t_o$

in equation (2-15), we have

$$\underline{x}(K+1)T = \Phi(T)\ \underline{x}(KT) + B(T)\ \underline{w}\ (KT) + D(T)$$
$$+ E(T)\ U_A(KT) \qquad (2\text{-}21)$$

where $\Phi(T)$, B(T), D(T) AND E(T) are given by equations (2-16), (2-17), (2-18) and (2-19) with $t - t_o$ replaced by T.

Equation (2-21) is a set of first-order difference equations and therefore is the discrete time model of the river system with aeration control only.

2-2b <u>Artificial aeration plus effluent discharge controls</u>

Equation (2-6) can be written as

$$\dot{\underline{x}}(t) = A(t)\ \underline{x}(t) + b\underline{w}(t) + d + e\underline{u}(t) \qquad (2\text{-}22)$$

where

$$A(t) = \begin{bmatrix} -\left(a_1 + \dfrac{Q + \bar{Q}_E}{V_m} + U_E(t)\right) & -a_2 \\ 0 & -\left(a_2 + \dfrac{Q + \bar{Q}_E}{V_m} + U_E(t)\right) \end{bmatrix}$$

$$b = \begin{bmatrix} \dfrac{Q}{V_m} & 0 \\ 0 & \dfrac{Q}{V_m} \end{bmatrix}$$

$$d = \begin{bmatrix} a_1 C_s - D_B + \dfrac{\bar{Q}_E C_E}{V_m} \\ L_A + \dfrac{\bar{Q}_E L_E}{V_m} \end{bmatrix}$$

$$e = \begin{bmatrix} 1 & C_E \\ 0 & L_E \end{bmatrix} \qquad \underline{U}(t) = \begin{bmatrix} U_A(t) \\ U_E(t) \end{bmatrix} \qquad \underline{w}(t) = \begin{bmatrix} C_i(t) \\ L_i(t) \end{bmatrix}$$

Since A(t) is a time varying matrix, the derivation of the state transition matrix, using the Laplace transforms technique becomes complicated and in fact impossible since the control function $U_E(t)$ is still to be found. In order to simplify the derivation of the discrete-time model for the two controls case, using the Laplace transform technique, it will be assumed that, since $U_E(t)$ varies very slowly, $U_E(t)$ is constant over a sample interval $KT \leq t \leq (K + 1)T$. Therefore equation (2-22) can be written as

$$\dot{\underline{x}}(t) = A(KT)\, \underline{x}(t) + b\underline{w}(t) + d + e\underline{u}(t) \qquad (2\text{-}23)$$

$$\text{for } KT \leq t \leq (K + 1)T$$

where

$$A(KT) = A(t)\Big|_{t = KT} \qquad \text{and } K = 0, 1, 2 \ldots.$$

A(KT) is now a constant matrix between sampling instants, hence a set of difference equations can be obtained using the Laplace transform technique as in section 2-2a. The discrete-time model is

$$\underline{x}(K + 1)T = \Phi(K,T)\, \underline{x}(KT) + B(K,T)\, \underline{w}(KT) + D(K,T) + E(K,T)\, \underline{u}(KT) \qquad K = 0, 1, 2 \ldots. \qquad (2\text{-}24)$$

where

$$\phi(K,T) = \begin{bmatrix} e^{-c_1T} & \dfrac{-a_2}{a_2 - a_1}[e^{-c_1T} - e^{-c_2T}] \\ 0 & e^{-c_2T} \end{bmatrix}$$

$$B(K,T) = \begin{bmatrix} (\dfrac{Q}{V_m})(1 - e^{-c_1T}) & \dfrac{-a_2}{c_1c_2}[1 - \dfrac{c_2e^{-c_1T}}{a_2 - a_1} + \dfrac{c_1e^{-c_2T}}{a_2 - a_1}] \\ 0 & \dfrac{(\frac{Q}{V_m})(1 - e^{-c_2T})}{c_2} \end{bmatrix}$$

$$D(K,T) = \begin{bmatrix} \dfrac{\phi_{x_1}(1 - e^{-c_1T})}{c_1} - \dfrac{a_2\phi x_2}{c_1c_2}[1 - \dfrac{c_2e^{-c_1T}}{a_2 - a_1} + \dfrac{c_1e^{-c_2T}}{a_2 - a_1}] \\ \dfrac{\phi_{x_2}}{c_2}[1 - e^{-c_2T}] \end{bmatrix}$$

$$E(K,T) = \begin{bmatrix} \dfrac{(1 - e^{-c_1T})}{c_1} & \dfrac{C_E(1 - e^{-c_1T})}{c_1} - \dfrac{a_2L_E}{c_1c_2} \\ & [1 - \dfrac{c_2e^{-c_1T}}{a_2 - a_1} + \dfrac{c_1e^{-c_2T}}{a_2 - a_1}] \\ 0 & \dfrac{L_E}{c_2}[1 - e^{-c_2T}] \end{bmatrix}$$

$$c_1 = a_1 + \frac{Q + \bar{Q}_E}{V_m} + U_E(K)$$

$$c_2 = a_2 + \frac{Q + \bar{Q}_E}{V_m} + U_E(K)$$

2.3 Controllability of Discrete-Time Models

This Chapter will be concluded with a discussion of the controllability of the models derived earlier.

THEOREM [12]

Given a continuous system

$$\dot{\underline{X}} = A\,\underline{X} + B\,\underline{m}$$

where

$\underline{X}$ is an n dimensional state vector

$\underline{m}$ is an r dimensional control vector

A is an nxn system matrix

B is an nxr control matrix

If A and B are time invariant matrices then the system is completely controllable if and only if the controllability matrix

$$P = [B \mid AB \mid A^2B \mid \ldots\ldots\ldots\ldots \mid A^{n-1}]$$

has n linearly independent column vectors, or alternatively, if the matrix P has a rank n.

The concept of controllability for discrete-time systems is similar to that for continuous systems except that the state equations are difference equations.

The condition for complete controllability of a digital system is such that given the system equation

$$\underline{X}\,(K + 1)T = A(T)\,\underline{X}\,(KT) + B(T)\,\underline{U}\,(KT)$$

The matrix

$$[s_o(T) \mid S_1(T) \mid S_2(T) \mid \ldots\ldots\ldots\ldots \; S_{n-1}(T)]$$

Where

$$s_o(T) = A(-T)B(T)$$

$$S_1(T) = A(-2T)B(T)$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$S_{n-1}(T) = A(-nT)B(T)$$

and T is the sample period.

have n linearly independent column vectors. In other words, given an initial state $\underline{X}_o$ there exists a set of controls $\underline{U}(KT)$ K = 0, 1, 2 ...... n-1 that will transfer the system to any desired final state in a given finite number of sample periods.

A look at the $\Phi(T)$ and $E(T)$ matrices of the system model for the aeration control case immediately reveals that the system is not completely controllable, since the control $U_A(KT)$ has no effect on the state variable X_2.

This is not a set back since the main objective is to control the concentration of dissolved oxygen in the river. Therefore the X_2(BOD) variable will be treated as a disturbance term in the design of a controller for the system. Only the controllable part of the model will be used in the design of the controller.

For the aeration plus effluent discharge case however, the $\Phi(K,T)$ matrix varies between sample intervals since it is a function of $U_E(K)$, the effluent discharge control. However, the system is controllable since a set of controls $[U_A(K)\ U_E(K)]$ can be found that will transfer the system from any initial state to any desired final state in any desired number of sample periods.

It will also be noticed that the controllability matrix for discrete time systems is a function of the sample period. Thus the assertion made above about the controllability of the system is on the assumption that a suitable sample period can be found such that the linear independence of the column vectors of the controllability matrix is not violated. The choosing of the sample period for the systems will be discussed in the next Chapter.

CHAPTER 3

DESIGN OF DIGITAL CONTROLLERS FOR WATER QUALITY

3.1 Introduction

In this chapter, the design of digital controllers to maintain the concentration of dissolved oxygen at a prescribed level using pole assignment technique with output feedback is discussed. The discrete time models derived in chapter 2 are used in the design.

Consider the water quality control system shown in figure 3.1 where the process to be controlled is given by

$$\underline{x}(K+1)T = \Phi(T)x(KT) + E(T)\underline{U}(KT) + D(T) + B(T)\underline{w}(KT) \tag{3-1}$$

where

$\underline{x}$ is the state variable vector

Φ is the system matrix

$\underline{U}$ is the control vector

E is the control matrix

D(T) and $B(T)\underline{w}(KT)$ are disturbance terms.

$K = 0,1,2 \ldots.$

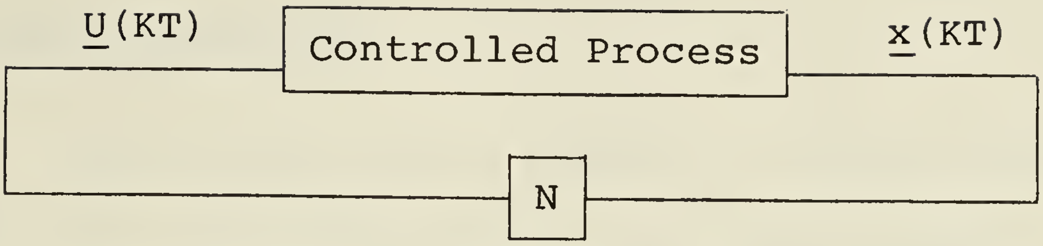


Figure 3.1 Water Quality Control System

Since $\underline{U}(KT) = N\underline{x}(KT)$, neglecting the disturbance terms, equations (3-1) can be written as

$$\underline{x}(K+1)T \quad = \quad \Phi(T)\underline{x}(KT) + E(T)N\underline{x}(KT)$$

or

$$\underline{x}(K+1)T \quad = \quad (\phi(T) + E(T)N)\ \underline{x}(KT) \tag{3-2}$$

Taking z transforms of equation (3-2), we have

$$z\underline{X}(z) - \underline{x}(0) \quad = \quad (\Phi(T) + E(T)N)\underline{X}(z) \tag{3-3}$$

Therefore

$$\underline{X}(z) \quad = \quad [zI = (\Phi(T) + E(T)N)]^{-1}\ \underline{x}(0) \tag{3-4}$$

where I is the unit matrix.

The poles of the closed loop system transfer matrix are given by the roots of

$$\det[zI = (\Phi(T) + E(T)N)] \quad = \quad 0 \tag{3-5}$$

The matrix N to locate these roots at any desired location within the unit circle in the z plane, can be determined by using any standard method of pole assignment [18, 19, 25].

3.2 Sampling Interval

As pointed out in chapter 2, since Φ and E in equation (3-5) are functions of the sampling period of the discrete time system, it is necessary to determine an appropriate sample interval to ensure controllability of the systems. For this purpose the sampling theorem will be invoked at this point.

Sampling theorem [12]

If a continuous signal contains no frequency higher than ω_c radians per unit time, the signal is completely characterized by the values of the signal measured at instants of time separated by

$$T = \pi/\omega_c \tag{3-6}$$

Case 1: Artificial aeration control

Using the values used by Ramar and Gourishankar [18, 19] viz

$$\left.\begin{array}{llllll} a_1 & = & 0.2 & a_2 & = & 0.32 \\ Q & = & 8.4 \times 10^4 & Q_E & = & 2.8 \times 10^4 \\ L_E & = & 20.2 & C_E & = & 2.0 \\ L_A & = & 0.0 & C_S & = & 11.0 \\ D_B & = & 1.0 & V_m & = & 15.1 \times 10^4 \end{array}\right\} \tag{3-7}$$

and replacing $t - t_o$ by T in equations (2-16), (2-17), (2-18) and (2-19) we have

$$\Phi(T) = \begin{bmatrix} e^{-0.9417T} & -2.67[e^{-0.9417T} - e^{-1.0617T}] \\ 0 & e^{-1.0617T} \end{bmatrix} \quad (3\text{-}8)$$

$$B(T) = \begin{bmatrix} 0.59(1-e^{-0.9417T}) & -0.178(1 - 8.85e^{-0.9417T} + 7.85e^{-1.0617T} \\ 0 & 0.52(1-e^{-1.0617T}) \end{bmatrix}$$

$$(3\text{-}9)$$

$$D(T) = \begin{bmatrix} 1.66(1-e^{-0.9417T}) - 1.187(1-8.85e^{-0.9417T} + 7.85e^{-1.0617T}) \\ 3.49(1-e^{-1.0617T}) \end{bmatrix} \quad (3\text{-}10)$$

$$E(T) = \begin{bmatrix} 1.062(1-e^{-0.9417T}) \\ 0 \end{bmatrix} \quad (3\text{-}11)$$

Noting that the open loop time constant of the variable x_1 in equation (3-8) is $\frac{1}{0.9417}$ or 1.0619 days and that the open loop system is overdamped, it is reasonable to specify the highest frequency contained in the continuous system response of the variable x_1 to be 4.0 radians per day. Therefore, substituting $\omega_c = 4.0$ radians per day into equation (3-6), the sampling period is

$$\frac{3.1416}{4.0} = 0.79 \text{ day}$$

This sample period can be used but a $T = 0.5$ day is more convenient (also more accurate). Substituting $T = 0.5$

into equations (3-8), (3-9), (3-10) and (3-11) we have

$$\Phi = \begin{bmatrix} 0.6245 & -0.0901 \\ 0 & 0.5881 \end{bmatrix} \tag{3-12}$$

$$B = \begin{bmatrix} 0.2215 & -0.0160 \\ 0 & 0.2140 \end{bmatrix} \tag{3-13}$$

$$D = \begin{bmatrix} 0.5166 \\ 1.4375 \end{bmatrix} \tag{3-14}$$

$$E = \begin{bmatrix} .3987 \\ 0 \end{bmatrix} \tag{3-15}$$

Substituting for Φ, B, D and E from equations (3-12), (3-13), (3-14) and (3-15), equation (2-21) for a sampling period of $T = 0.5$ becomes

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \begin{bmatrix} .6245 & -0.0901 \\ 0 & 0.5881 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0.2215 & -0.0160 \\ 0 & 0.2140 \end{bmatrix} \begin{bmatrix} C_i(K) \\ L_i(K) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.5166 \\ 1.4375 \end{bmatrix} + \begin{bmatrix} 0.3987 \\ 0 \end{bmatrix} U_A(K) \tag{3-16}$$

Case 2: Artificial aeration plus effluent discharge as controls

It was shown in chapter 2 that the Φ and E matrices for this case are dependent on the effluent discharge control. Consequently the system matrix Φ varies from one sample period to the next. A precise computation of the optimum sampling period would be difficult. However, since we can expect the response of the variable x_1 to contain higher harmonics compared to the single control case, a sampling period less than 0.5 day would seem desirable. For convenience $T = 0.25$ day will be used. The suitability of the choice is confirmed by simulation results presented later in this chapter.

Substituting $T = 0.25$ and equation (3-7) into equation (2-24) and noting the fact that the nominal value of Q_E; $\bar{Q}_E = 2.8 \times 10^4$ we have

$$\underline{x}(K+1) = \Phi(K)\underline{x}(K) + B(K)\underline{w}(K) + D(K) + E(K)\underline{U}(K) \qquad (3\text{-}17)$$

where

$$\Phi(K) = \begin{bmatrix} 0.7902e^{-0.25U_E(K)} & -0.0075e^{-0.25U_E(K)} \\ 0 & 0.7669e^{-0.25U_E(K)} \end{bmatrix} \qquad (3\text{-}18)$$

$$B(K) = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \qquad (3\text{-}19)$$

where

$$\psi_{11} = \frac{0.556(1 - 0.7902e^{-0.25U_E(K)}}{0.9417 + U_E(K)}$$

$$\psi_{12} = \frac{-0.178[1 - (0.9731 + 0.942U_E(K))e^{-0.25U_E(K)}]}{(0.9417 + U_E(K))(1.0617 + U_E(K))}$$

$$\psi_{21} = 0$$

$$\psi_{22} = \frac{0.556[1 - 0.7669e^{-0.25U_E(K)}]}{1.0617 + U_E(K)}$$

$$D(K) = \begin{bmatrix} \dfrac{1.571[1 - 0.7902e^{-0.25U_E(K)}]}{0.9417 + U_E(K)} - \dfrac{1.187[1 - (0.9731 + 0.1942U_E(K))e^{-0.25U_E(K)}}{(0.9417 + U_E(K))(1.0617 + U_E(K)} \\[3ex] \dfrac{3.71(1 - 0.7669e^{-0.25U_E(K)})}{1.0617 + U_E(K)} \end{bmatrix} \tag{3-20}$$

$$E(K) = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \tag{3-21}$$

where

$$\sigma_{11} = \frac{[1 - 0.7902e^{-0.25U_E(K)}]}{[0.9417 + U_E(K)]}$$

$$\sigma_{12} = \frac{2(1 - 0.7902e^{-0.25U_E(K)})}{(0.9417 + U_E(K)} - \frac{6.4[1 - (0.9731 + 0.1942U_E(K))e^{-0.25U_E(K)}]}{[0.9417 + U_E(K)][1.0617 + U_E(K)]}$$

$$\sigma_{21} = 0$$

$$\sigma_{22} = \frac{20[1 - 0.7669e^{-0.25U_E(K)}]}{[1.0617 + U_E(K)]}$$

If $e^{-0.25U_E(K)}$ is expanded in a Taylor series and the first two terms are taken (this is acceptable if $U_E(K)$ is assumed to be small for all K then

$$e^{-0.25U_E(K)} = 1 - 0.25\, U_E(K) \tag{3-22}$$

Substituting equation (3-22) into equation (3-18), (3-19), (3-20) and (3-21) and assuming that $U_E(K)$ is small compared to 0.9417 for all K, the equations become

$$\Phi(K) = \begin{bmatrix} 0.7902 - 0.1976U_E(K) & -0.0075 + 0.0019U_E(K) \\ 0 & 0.7669 - 0.1917U_E(K) \end{bmatrix} \tag{3-23}$$

$$B(K) = \begin{bmatrix} 0.1239 + 0.1167U_E(K) & .0048 + 0.0087U_E(K) \\ 0 & .1221 + 0.1004U_E(K) \end{bmatrix} \tag{3-24}$$

$$D(K) = \begin{bmatrix} 0.3179 + 0.2711U_E(K) \\ 0.8148 + 0.6699U_E(K) \end{bmatrix} \tag{3-25}$$

$$E(K) = \begin{bmatrix} 0.2228 + 0.2098U_E(K) & 0.2734 + 0.1015U_E(K) \\ 0 & 4.3911 + 3.6112U_E(K) \end{bmatrix} \tag{3-26}$$

Equation (3-17) can therefore be written as

$$\underline{x}(K+1) = (\bar{\Phi} + \tilde{\Phi}(K))\ \underline{x}(K) + [\bar{B} + \tilde{B}(K)]\ \underline{w}(K) +$$

$$[\bar{D} + \tilde{D}(K)] + [\bar{E} + \tilde{E}(K)]\ \underline{U}(K) \tag{3-27}$$

where

$$\bar{\Phi} = \begin{bmatrix} 0.7902 & -0.0075 \\ 0 & 0.7669 \end{bmatrix}$$

$$\tilde{\Phi}(K) = \begin{bmatrix} -0.1976U_E(K) & 0.0019U_E(K) \\ 0 & -0.1917U_E(K) \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 0.1239 & 0.0048 \\ 0 & 0.1221 \end{bmatrix}$$

$$\tilde{B}(K) = \begin{bmatrix} 0.1167U_E(K) & 0.0087U_E(K) \\ 0 & 0.1004U_E(K) \end{bmatrix}$$

$$\bar{D} = \begin{bmatrix} 0.3179 \\ 0.8148 \end{bmatrix} \qquad \tilde{D}(K) = \begin{bmatrix} 0.2711U_E(K) \\ 0.6699U_E(K) \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} 0.2228 & 0.2734 \\ 0 & 4.3911 \end{bmatrix}$$

$$\tilde{E} = \begin{bmatrix} 0.2098U_E(K) & 0.1015U_E(K) \\ 0 & 3.6112U_E(K) \end{bmatrix}$$

Equation (3-27) is the discrete time model for the artificial aeration plus effluent discharge controls case for a sampling period of 0.25 day.

3.3 Digital Controller for Artificial Aeration as the Only Control

We shall now design the controller N for the single control case which will take care of deviations from the nominal steady state values of the state variables. These values are

$$\left.\begin{aligned} \bar{x}_1 &= 6.0 \\ \bar{x}_2 &= 6.61 \\ \bar{C}_i &= 10.0 \\ \bar{L}_i &= 6.0 \\ \bar{U}_A &= 0.55 \end{aligned}\right\} \qquad (3\text{-}28)$$

It was mentioned earlier that the input aeration has no effect on the BOD output of the system. Now the controllable part of the model (equation 3-16) is

$$\begin{aligned} x_1(K+1) = {} & 0.6245x_1(K) - 0.0972x_2(K) + 0.2215C_i(K) \\ & - 0.016L_i(K) + 0.5116 + 0.3987U_A(K) \end{aligned} \qquad (3\text{-}29)$$

If the desired output DO is the nominal steady state value of x_1; $\bar{x}_1$, the error between the desired output and the actual output of the open loop system is

$$\begin{aligned} E_1(K+1) = {} & 0.6245E_1(K) - 0.0972E_2(K) + 0.2215\delta C_i(K) \\ & - 0.016\delta L_i(K) + 0.3987\delta U_A(K) \end{aligned} \qquad (3\text{-}30)$$

where

$$E_1 = x_1 - \bar{x}_1$$

$$\left.\begin{aligned} E_2 &= x_2 - \bar{x}_2 \\ \delta C_i &= C_i - \bar{C}_i \\ \delta L_i &= L_i - \bar{L}_i \\ \delta U_A &= U_A - \bar{U}_A \end{aligned}\right\} \tag{3-30a}$$

In order to obtain a closed loop system with acceptable steady state error, a third variable E_3 is introduced. The state equation is

$$E_3(K+1) = E_3(K) - E_1(K) \tag{3-31}$$

This is equivalent to the integral control for continuous systems.

The river system is now represented by the augmented system.

$$\begin{bmatrix} E_1(K+1) \\ E_3(K+1) \end{bmatrix} = \begin{bmatrix} 0.6245 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} E_1(K) \\ E_3(K) \end{bmatrix} - \begin{bmatrix} 0.0972 \\ 0 \end{bmatrix} E_2(K) +$$

$$\begin{bmatrix} 0.2215 & -0.016 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta C_i(K) \\ \delta L_i(K) \end{bmatrix} + \begin{bmatrix} 0.3987 \\ 0 \end{bmatrix} \delta U_A(K) \tag{3-32}$$

$$\text{Let } \delta U_A(K) = N_1 E_1(K) + N_2 E_3(K) \tag{3-33}$$

Substituting equation (3-33) into equation (3-32) we have

$$\begin{bmatrix} E_1(K+1) \\ E_3(K+1) \end{bmatrix} = \begin{bmatrix} (0.6245 + 0.3987N_1) & 0.3987N_2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} E_1(K) \\ E_3(K) \end{bmatrix} -$$

$$\begin{bmatrix} 0.0972 \\ 0 \end{bmatrix} E_2(K) + \begin{bmatrix} 0.2215 & -0.016 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta C_i(K) \\ \delta L_i(K) \end{bmatrix} \qquad (3\text{-}33)$$

The characteristic equation of this system is given by

$$\det \begin{bmatrix} z - (0.6245 + 0.3987N_1) & -0.3987N_2 \\ 1 & (z-1) \end{bmatrix} = 0$$

that is

$$z^2 - (1.6245 + 0.3987N_1)z + (0.6245 + 0.3987N_1 + 0.3987N_2) = 0 \qquad (3\text{-}34)$$

Noting that the open loop system is overdamped, let the closed loop poles of the system represented by equation (3-33) be located within the unit circle in the z plane at

$$\left.\begin{aligned} z_1 &= 0.3094 + 0.5488j \\ z_2 &= 0.3094 = 0.5488j \end{aligned}\right\} \qquad (3\text{-}35)$$

This choice of closed loop poles was arrived at after several computer simulation runs with different pole locations. The characteristic equation of the closed loop system for this pole location is therefore

$$z^2 - 0.6188z + 0.3969 = 0 \qquad (3\text{-}36)$$

Equations (3-34) and (3-36) both represent the characteristic equation of the closed loop system, therefore comparing coefficients, N_1 and N_2 are found to be

$$N_1 = -2.5224$$
$$N_2 = 1.9516 \qquad (3\text{-}37)$$

Substituting N_1 and N_2 into equation (3-33) the incremental control law is

$$\delta U_A(K) = -2.5224E_1(K) + 1.9516E_3(K) \qquad (3\text{-}38)$$

3.4 Implementation of the Artificial Aeration Control Scheme and Simulation Results

The practical implementation of this control law should not permit negative values of the control function, $U_A(K) = \delta U_A(K) + \overline{U}_A$, as this would mean removal of dissolved oxygen from the river. It therefore follows that when such conditions occur, that is $U_A(K)$ is negative, the control action of the controller is not desired and the controller should be turned off until the DO level drops down to the desired level.

This constraint means that the control function can only take on positive values (including zero) and should be set equal to zero if the function is negative.

This objective can be achieved by inserting a limiter between the controller and the zero order hold in the schematic diagram of figure 2.1. This arrangement is shown in figure 3.2. This scheme was simulated on a digital computer and the simulation results are shown in figure 3.6(a, b, and c) for an assumed input DO and BOD profiles shown in figure 3.3(a and b). The DO and BOD profiles are the

staircase approximation of those used by Ramar and Gourishankar [18, 19].

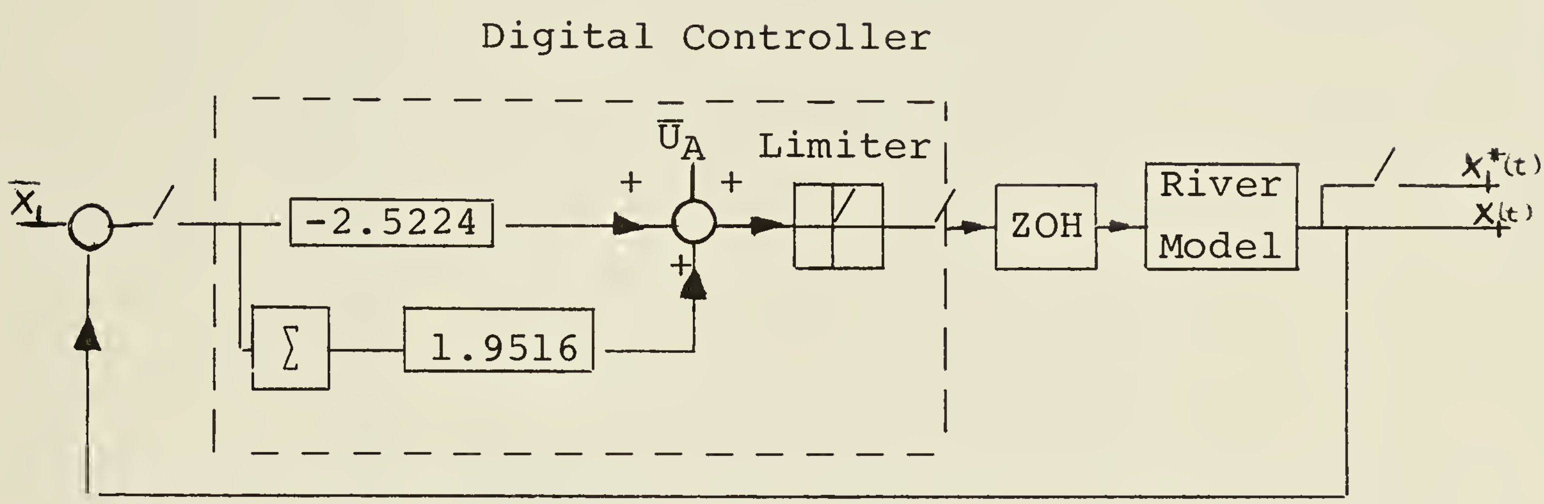
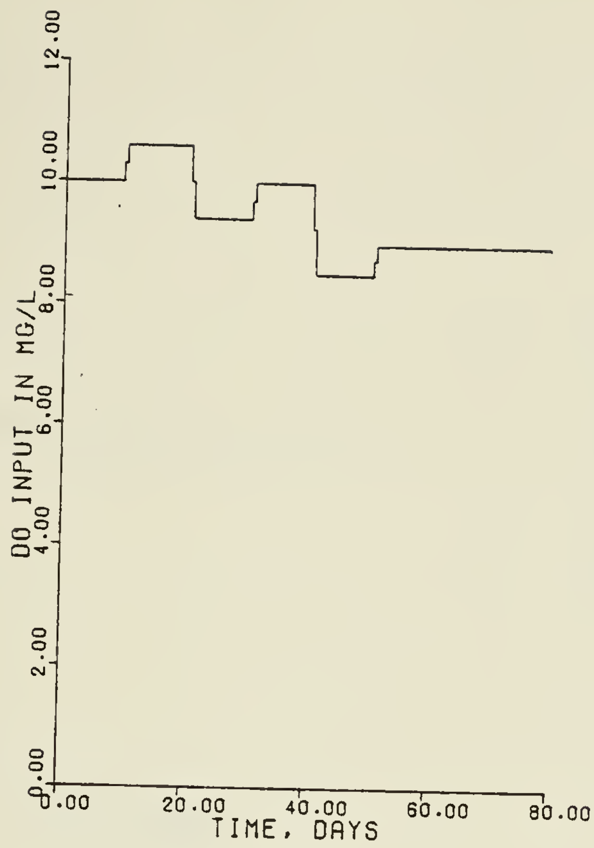


Figure 3.2 Artificial Aeration Control Scheme

3.5 Digital Controller for Artificial Aeration Plus Effluent Discharge as Controls

The design of a controller for the two controls case is not as straight forward as for the single control case since the state transition matrix Φ is a function of the effluent discharge control. The strategy we will use is to split the model represented by equation (3-27) into a fixed part and a variable part and design the controller using the fixed part of the model. The effects of the variable part of the model can then be minimized by carefully choosing the gain matrix N of the controller using the same method used in the last section.

Using this strategy, equation (3-27) can be written as the sum of two equations.



(a)

(b)

Figure 3.3 Assumed Upstream DO and BOD Profiles

(a) DO

(b) BOD

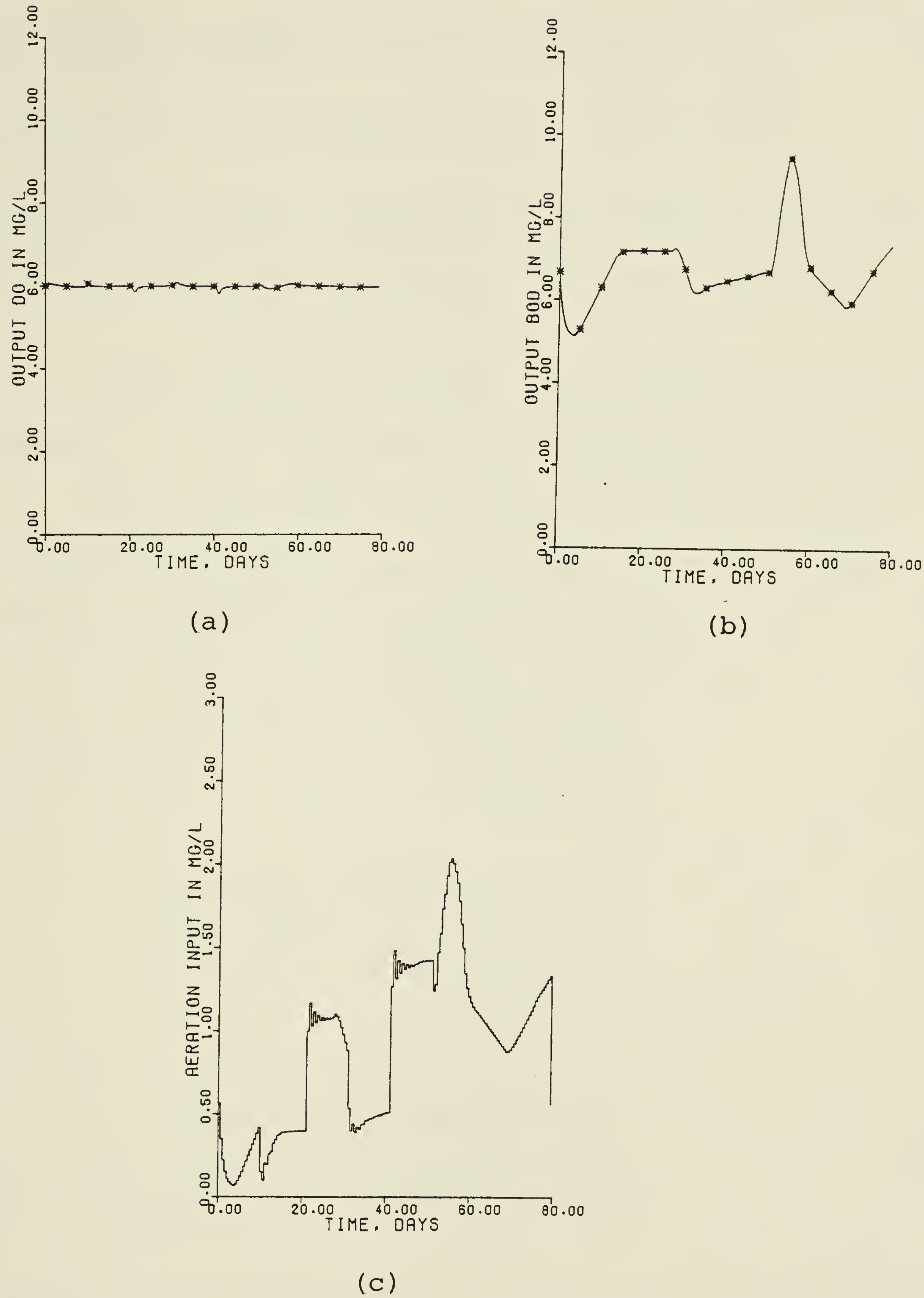


Figure 3.4 Discrete-Time System Response (Constant-Parameters) Aeration Control Case

(a) Output DO

(b) Output BOD

(c) Artificial Aeration

$$\bar{x}_f(K+1) = \bar{\Phi}\underline{x}(K) + \bar{E}\underline{U}(K) + \bar{B}\underline{w}(K) + \bar{D} \tag{3-39}$$

$$\tilde{\underline{x}}(K+1) = \tilde{\Phi}\underline{x}(K) + \tilde{E}\underline{U}(K) + \tilde{B}\underline{w}(K) + \tilde{D} \tag{3-40}$$

Equation (3-39) is the fixed part of the model represented by equation (3-27) and equation (3-40) is the variable part.

Using the pole assignment technique and considering only equation (3-39) the controller is designed as follows:

If we once again assume that the desired output DO is the nominal value of x_1; $\bar{x}_1$, then the error between the desired output $\bar{x}_1$ and the actual output is

$$\begin{bmatrix} E_1(K+1) \\ E_2(K+1) \end{bmatrix} = \begin{bmatrix} 0.7902 & -0.0075 \\ 0 & 0.7669 \end{bmatrix} \begin{bmatrix} E_1(K) \\ E_2(K) \end{bmatrix} + \begin{bmatrix} 0.2228 & 0.2734 \\ 0 & 4.3911 \end{bmatrix} \begin{bmatrix} \delta U_A(K) \\ U_E(K) \end{bmatrix} + \begin{bmatrix} 0.1239 & 0.0048 \\ 0 & 0.1221 \end{bmatrix} \begin{bmatrix} \delta C_i(K) \\ \delta L_i(K) \end{bmatrix} \tag{3-41}$$

Where E_1, E_2, δC_i, δL_i and δU_A are defined by equation (3-30a).

In order to maintain an acceptable steady state error, a third variable E_3, defined by equation (3-31) is introduced. The river system is therefore represented by the augmented system

$$\begin{bmatrix} E_1(K+1) \\ E_2(K+1) \\ E_3(K+1) \end{bmatrix} = \begin{bmatrix} 0.7902 & -0.0075 & 0 \\ 0 & 0.7669 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_1(K) \\ E_2(K) \\ E_3(K) \end{bmatrix} +$$

$$\begin{bmatrix} 0.2228 & 0.2732 \\ 0 & 4.3911 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta U_A(K) \\ \\ U_E(K) \end{bmatrix} +$$

$$\begin{bmatrix} 0.1239 & 0.0048 \\ 0 & 0.1221 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta C_i(K) \\ \\ \delta L_i(K) \end{bmatrix} \tag{3-42}$$

Let

$$\underline{U}(K) = \begin{bmatrix} N_1 & N_2 \\ N_3 & N_4 \end{bmatrix} \begin{bmatrix} E_1(K) \\ E_3(K) \end{bmatrix} \tag{3-43}$$

Substituting for $\underline{U}(K)$ in equation (3-42) we have

$$\begin{bmatrix} E_1(K+1) \\ \\ E_2(K+1) \\ \\ E_3(K+1) \end{bmatrix} = \begin{bmatrix} (0.7902 + 0.2228N_1 & -0.0075 & 0.2228N_2 + \\ + 0.2734N_3) & & 0.2734N_4 \\ 4.3911N_3 & 0.7669 & 4.3911N_4 \\ & & \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_1(K) \\ \\ E_2(K) \\ \\ E_3(K) \end{bmatrix} +$$

$$\begin{bmatrix} 0.1239 & 0.0048 \\ 0 & 0.1221 \\ 0 & 0 \end{bmatrix} \delta \underline{w}(K) \tag{3-44}$$

The characteristic equation is therefore given by

$$\begin{aligned} & z^3 - (2.5571 + 0.2228N_1 + 0.2734N_3)z^2 + \\ & \quad (2.1631 + 0.3937N_1 + 0.2228N_2 + 0.5160N_3 + 0.2734N_4)z - \\ & \quad (0.606 + 0.1709N_1 + 0.1709N_2 + 0.2426N_3 + 0.2426N_4) = 0 \end{aligned} \tag{3-45}$$

The closed loop poles of the discrete-time system are chosen as

$$\begin{aligned} z_1 &= 0.7525 \\ z_{2,3} &= 0.5207 \pm 0.5971 \end{aligned} \tag{3-46}$$

In making the above choice of pole location we have arbitrarily chosen the real pole of the system to be at the point S = -1.1 in the S plane. This maps into the point z = 0.7525 in the z plane. The complex poles were then chosen after several simulation runs.

The characteristic equation for this pole location can therefore be written as

$$z^3 - 1.7939z^2 + 1.4114z - 0.4723 = 0 \tag{3-47}$$

Equation (3-45) and (3-47) are the same, therefore comparing coefficients and solving the resulting equations give

$$N_1 = -2.4401 + 5.2678N_4$$
$$N_2 = 2.7977 - 0.5934N_4 \qquad (3\text{-}48)$$
$$N_3 = -0.8030 - 4.2929N_4$$

Since N_1, N_2 and N_3 are functions of N_4, N_4 should be chosen such that the variations of the output DO due to parameter variations are as small as possible.

The derivations of sensitivity functions for discrete-data systems are available in the literature [10, 13, 20]. A simple method for investigating the sensitivity of the output vector of a multivariable system due to parameter variation proposed by Soliman and Kevorkian [20] is used in finding a suitable value of N_4.

Soliman and Kevorkian [20] defined the sensitivity of the output vector of a multivariable sample data system to small parameter variations as the ratio of the norm of the output vector deviations in the closed loop realization of the system to the norm of the output deviations in the open loop realization of the system. Thus the sensitivity of each output of the multivariable system to small parameter variations can be written as

$$S_{n_1}(KT) = \left\| \frac{e_{c_1}(KT)}{e_{o_1}(KT)} \right\|$$

$$S_{n_2}(KT) = \left\| \frac{e_{c_2}(KT)}{e_{o_2}(KT)} \right\|$$

$\cdot$

$\cdot$

.

$$S_{n_K}(KT) = \left\| \frac{e_{c_K}(KT)}{e_{o_K}(KT)} \right\| \tag{3-49}$$

Where $S_{n_K}(KT)$ is the sensitivity of the Kth output of the multivariable discrete time system.

$e_{c_K}(KT)$ is the output vector deviation due to parameter variation in the closed loop realization of the system and $e_{o_K}(KT)$ is the output vector deviation due to parameter variation in the open loop realization of the system.

Noting that a good feedback design will ensure that the norm of the output vector deviation in the closed loop realization is much less than the norm of the output vector deviation in the open loop realization, that is

$$\begin{gathered} \left\| e_{c_1}(KT) \right\| << \left\| e_{o_1}(KT) \right\| \\ \left\| e_{c_2}(KT) \right\| << \left\| e_{o_2}(KT) \right\| \\ \vdots \\ \text{etc.} \end{gathered} \tag{3-50}$$

The performance of the two controllers in a closed loop realization of a system for the same controlled process could be investigated by simply comparing the norm of the output deviation due to parameter variation. The objective in our case is to find the value of N_4 that will minimize the variations of the output DO.

A graph of the norm of the deviation of the output DO from the desired level for various values of N_4 will indicate the value of N_4 for which the norm of this deviation is minimum. This graph is shown in figure 3.5. From the graph the value of N_4 is found to be -0.1910.

Substituting for N_4 in equation (3-48) the gain matrix for the controller becomes

$$\begin{bmatrix} -3.4462 & 2.9110 \\ 0.0169 & -0.1910 \end{bmatrix}$$

Therefore the control law is

$$\begin{aligned} \delta U_A(K) &= -3.4462E_1(K) + 2.9110E_3(K) \\ U_E(K) &= 0.0169E_1(K) - 0.1910E_3(K) \end{aligned} \qquad (3\text{-}51)$$

From $U_E(K)$, the quantity of effluent discharged into the river is

$$Q_E(K) = 15.1 \times 10^4(U_E(K) + \bar{U}_E \qquad (3\text{-}52)$$

For reasons stated in section 3.4, $U_A(K)$ and $Q_E(K)$ can not take on negative values, therefore the control law would have to be nonlinear. The nonlinear control law is

$$\begin{aligned} U_A(K) &= Y_1 \quad ; \quad Y_1 \geq 0 \\ U_A(K) &= 0 \quad ; \quad Y_1 < 0 \\ U_E(K) &= Y_2 \quad ; \quad Y_2 \geq 0 \\ U_E(K) &= 0 \quad ; \quad Y_2 < 0 \end{aligned} \qquad (3\text{-}53)$$

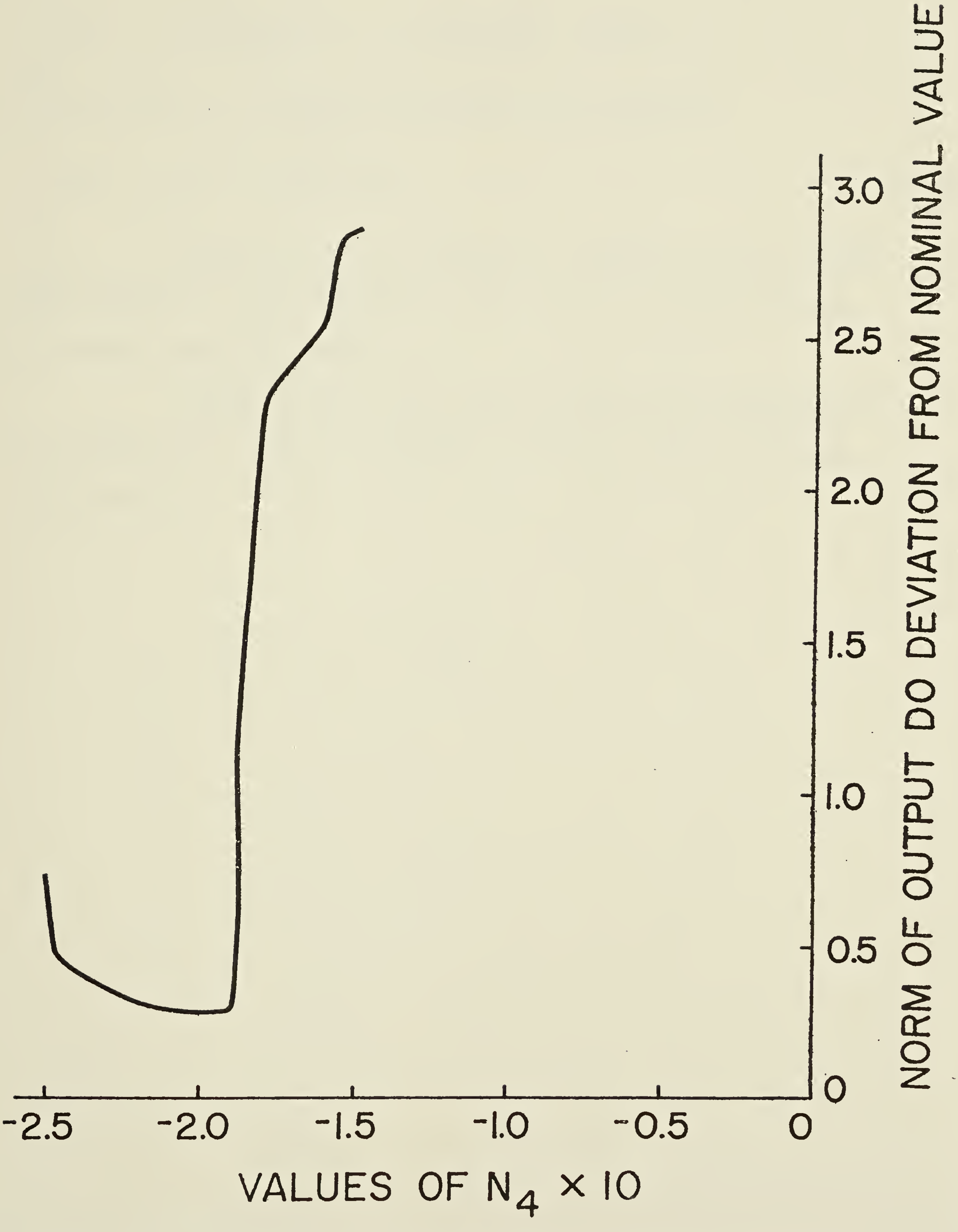


Figure 3.5 Norm of the Output DO Deviations versus N_4

where

$$Y_1 = -3.4462E_1(K) + 2.9110E_3(K) + \bar{U}_A(K)$$

$$Y_2 = 0.0169E_1(K) - 0.1910E_3(K) + \bar{U}_E(K)$$

The quantity of effluent discharge is therefore

$$Q_E = 15.1 \times 10^4 \times Y_2 \tag{3-54}$$

Figure 3.6 is a schematic diagram of the closed loop discrete time system for the aeration plus effluent discharge controls case.

Figure 3.7(a, b, c, and d) show the response of the system for input DO and BOD profiles shown in figure 3.3(a and b).

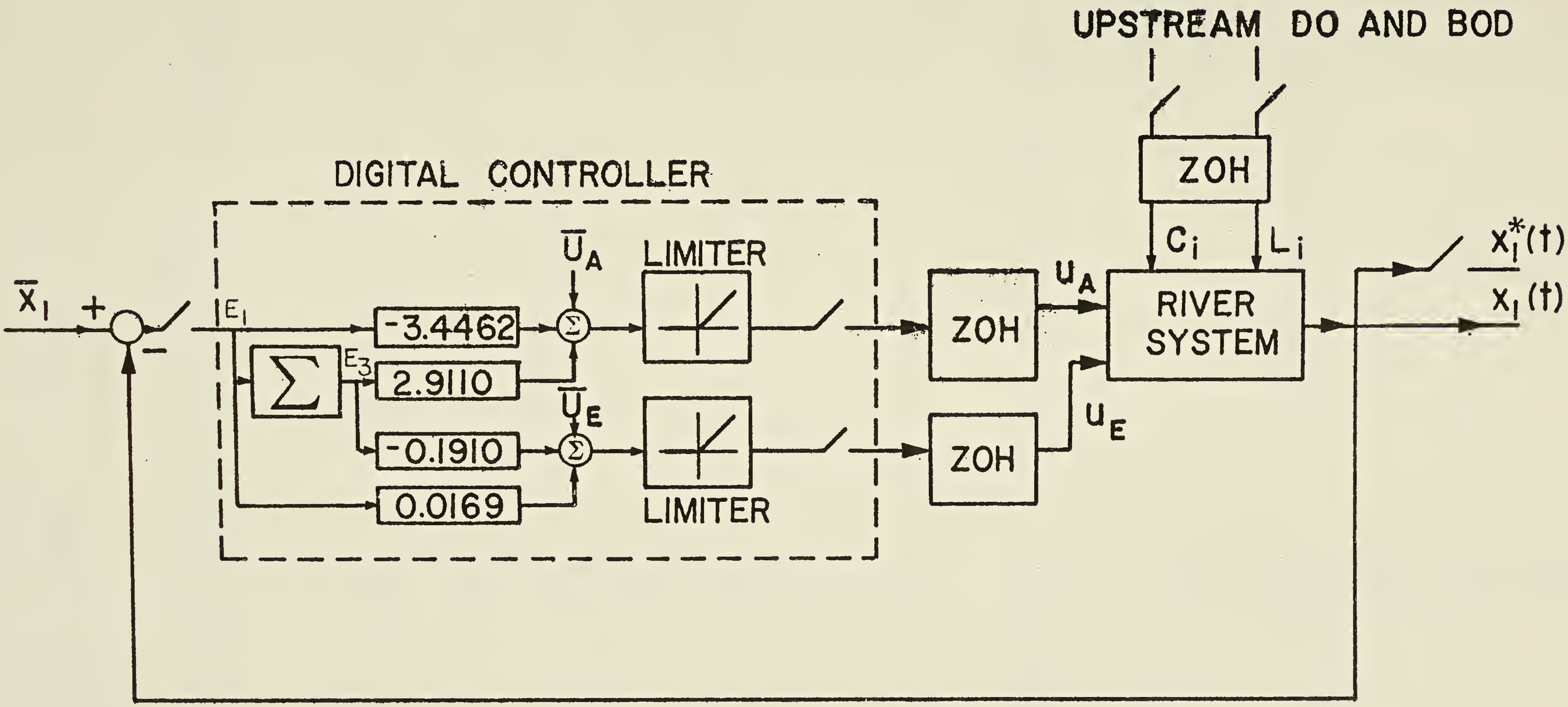


Figure 3.6 Artificial Aeration plus Effluent Discharge Control Scheme

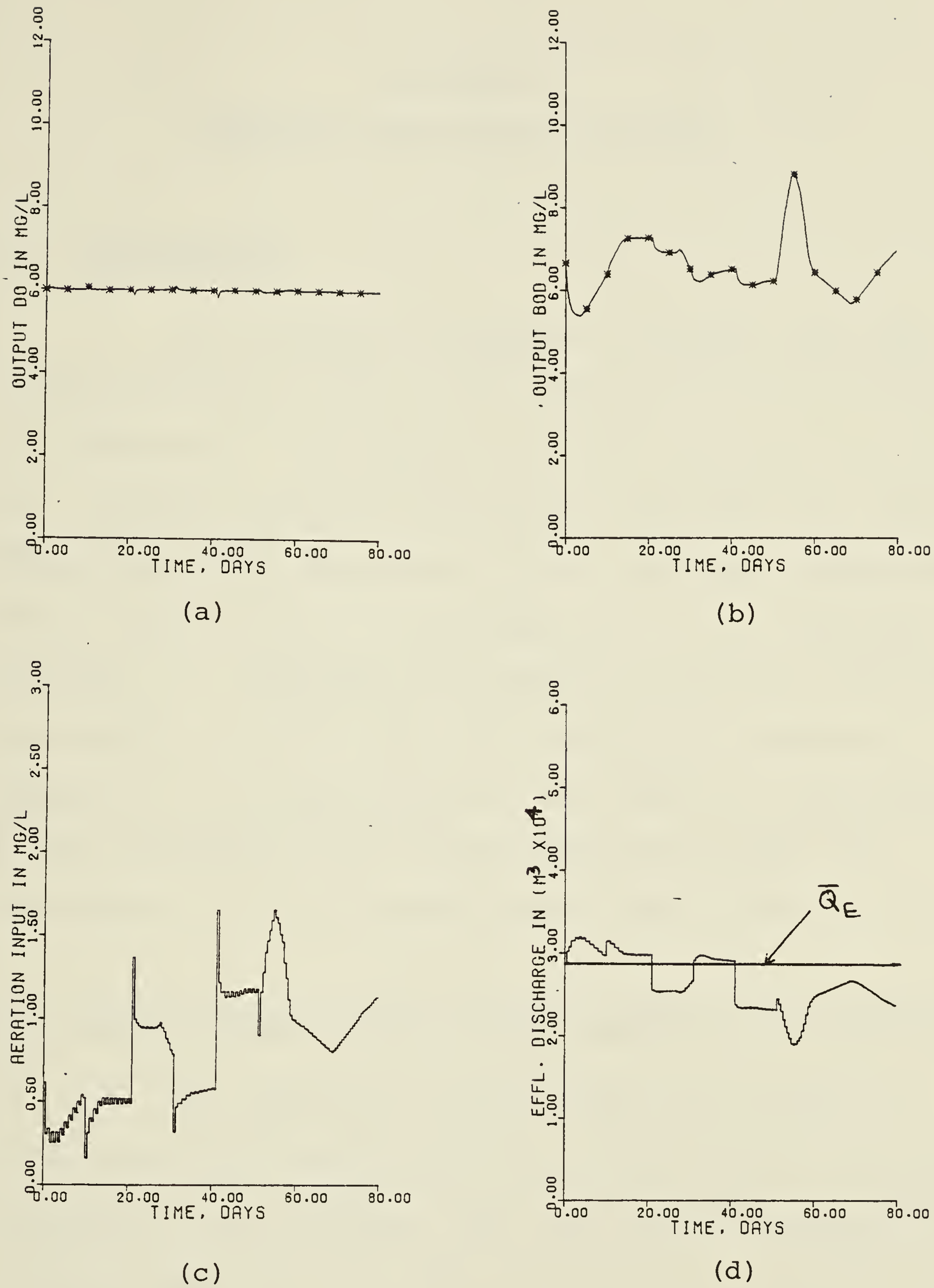


Figure 3.7 Discrete-Time System Response (Constant-Parameters) Aeration Plus Effluent Discharge Controls Case

(a) Output DO (b) Output BOD

(c) Artificial Aeration (d) Effluent Discharge

CHAPTER 4

SEASONAL TEMPERATURE VARIATION

4.1 Introduction

It is normally necessary to specify the range of weather conditions within which proper operation of systems are guaranteed. For example a look at the specifications for a typical electronic calculator shows that the operating temperature range is between 10°C to 40°C and outside this range a malfunction is to be expected. It is therefore necessary to investigate the effect of seasonal temperature variations on the discrete-time controllers designed in chapter 3. An investigation of the effect of temperature on the continuous time systems designed by Ramar and Gourishankar is also carried out in this chapter.

4.2 Relationships Between System Parameters and Temperature

The controllers designed for maintaining the level of DO in a single reach of a river system have been designed on the assumption that the system parameters a_1, a_2, C_S, D_B in equation (2-1) are all constants. In 1961, a committee on sanitation engineering research in the U.S.A. [2] published a report which establishes the relationship between water temperature and stream reaeration, and since then some researchers have been taking the effect of temperature variation on the river system parameters into account in

their analysis. [3, 25] Young and Beck [25], in assessing the suitability of their model suggested that a term dependent on temperature and sunlight conditions be added so that observed data could be correlated with values obtained by solving the system equations. Davidson and Bradshaw [3] have also analyzed the effect of water temperature on the DO and BOD profiles of a river system using the following temperature dependent parameters.

$$\begin{aligned} a_1(\tau) &= a_1(20°C)e^{0.025\tau°C} \\ a_2(\tau) &= a_2(20°C)e^{0.0464\tau°C} \\ C_S(\tau) &= C_S(20°C)e^{-0.021\tau°C} \\ D_B(\tau) &= \beta(25 - 0.028(\tau°C - 30)^2) \end{aligned} \qquad (4\text{-}1)$$

Where β is a constant.

$\tau°C$ is the temperature of the water in degrees celsius

$a_1(20°C)$ is the reaeration constant at 20°C

$a_2(20°C)$ is the BOD decay rate at 20°C

$C_S(20°C)$ is the saturation value of DO at 20°C

We will use these relationships in this investigation.

Since data for the water temperature for which Young and Beck [25] calculated the values for the various parameters are not available, a seasonal temperature profile is developed using the 1974 average monthly surface temperature for Great Britain. The average monthly temperatures for the various stations and the overall average for the twelve months of 1974 is shown in Table 1.

TABLE 1

AVERAGE MONTHLY TEMPERATURE FOR GREAT BRITAIN AS REPORTED IN THE

MONTHLY CLIMATIC DATA FOR THE WORLD BULLETIN

STATION	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEP	OCT	NOV	DEC
Lerwick	5.1	4.6	4.5	6.4	8.1	10.0	11.0	12.0	10.1	6.6	5.5	4.3
Stornway	6.2	5.3	5.7	7.5	9.7	11.1	12.0	12.9	10.1	7.7	5.9	5.7
Tiree	6.7	6.1	5.8	8.2	10.1	11.8	12.7	13.5	11.3	8.9	6.8	7.1
Aberdeen	4.7	4.7	4.3	6.5	9.4	11.7	13.0	13.7	10.3	7.2	5.5	5.6
Edinburgh	6.0	5.5	5.1	6.5	10.9	12.8	14.1	14.3	11.1	7.5	5.7	6.7
Eskdalemuir	3.9	3.4	3.4	5.7	8.7	10.7	12.1	12.0	9.0	5.8	4.3	5.4
Leeming	5.3	5.5	5.3	6.4	10.9	12.9	14.7	14.9	11.9	7.5	5.9	8.1
Belfast	5.8	5.3	5.5	8.3	10.3	12.5	13.8	14.3	10.7	7.5	5.5	6.8
Valey	7.6	6.8	6.6	9.2	11.0	13.9	14.5	15.0	12.8	9.3	7.8	8.7
Manchester	6.1	5.8	6.2	8.9	11.3	13.8	14.7	15.1	11.9	7.7	6.5	8.1
Birmingham	5.6	5.5	5.3	7.5	10.7	13.5	15.4	14.5	11.7	7.5	6.3	7.9
Waddington	5.3	5.3	5.3	7.1	10.7	13.3	15.1	15.4	12.1	7.6	6.1	7.5
Gorleston	5.9	3.4	5.3	6.9	10.5	13.0	15.5	16.2	13.2	8.1	7.1	7.2
Kew	6.9	6.1	6.1	8.9	11.4	14.5	16.1	15.8	12.9	8.1	7.7	8.7
London	6.3	5.7	5.7	8.3	10.7	14.2	15.3	15.3	12.5	7.5	7.3	7.9
Grawley												
Glamorgan	7.0	6.3	5.8	8.7	10.7	13.7	14.8	14.8	12.2	8.3	7.6	8.3
Bournemouth	7.3	6.3	6.4	8.5	10.9	14.3	15.7	15.2	12.7	8.1	8.2	8.7
Plymouth	8.4	7.2	7.0	9.5	10.8	14.1	14.9	14.9	12.6	9.1	9.1	9.3
AVERAGE	6.1	5.5	5.5	7.7	10.4	12.9	14.2	14.4	11.6	7.8	6.6	7.3

J.C. Ward [23] had reported that seasonal temperature variations could be represented by a sine wave of the form

$$\tau = \alpha(\sin(b\gamma + \phi)) + \bar{\tau} \tag{4-2}$$

where

τ is the temperature in °C

α is a constant

$b = 360/365$ or 0.986 degrees/day

$\bar{\tau}$ is the average yearly temperature in °C, and

γ is the number of days starting from October 1. That is, $\gamma = 1$ for October 1. A sine wave was fitted to the overall average monthly temperature of Table 1 using the least square fit method (see appendix 1). The values of α, Φ and $\bar{\tau}$ are obtained as

$\alpha = -4.3$

$\phi = -23.1°$

$\bar{\tau} = 9.19°C$

The equation of the temperature profile is

$$\tau°C = -4.3(\sin(0.986\gamma - 23.1)) + 9.19°C \tag{4.3}$$

Since Young and Beck collected their data in the summer months, it is reasonable to assume that the values of a_1, a_2, C_S and D_B used in their work are the values of these temperature dependent parameters evaluated at an average summer temperature. Thus if we assume that the summer starts on the first day of June and ends on the 31st day of August, an average summer temperature can be calculated thus

$$\tau_S^{\circ}C = \frac{-4.3}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \sin\theta d\theta + 9.19^{\circ}C$$

where

$$\theta_1 = (b\gamma_1 - 23.1)(\frac{\pi}{180}) \text{ radians}$$

$$\theta_2 = (b\gamma_2 - 23.1)(\frac{\pi}{180}) \text{ radians}$$

γ_1 and γ_2 are the number of days between October 1 and June 1 and October 1 and August 31, respectively.

$$\gamma_1 = 244 \text{ days}$$

$$\gamma_2 = 335 \text{ days}$$

$$\therefore \quad \theta_1 = 217.48^{\circ} \text{ or } 3.7957 \text{ radians}$$

$$\theta_2 = 307.21^{\circ} \text{ or } 5.3618 \text{ radians}$$

$$\therefore \quad \tau_S^{\circ}C = \frac{-4.3[-0.6047 - 0.7936]}{5.3618 - 3.7957} + 9.19$$

$$\therefore \quad \tau_S^{\circ}C = 13.03^{\circ}C \tag{4-4}$$

Substituting equation (4-4) into equation (4-1) and using $a_1(\tau) = 0.2$, $a_2(\tau) = 0.32$, $C_S(\tau) = 11.0$ and $D_B(\tau)=1.0$ we get

$$\begin{aligned} a_1(20^{\circ}C) &= 0.1444 \\ a_2(20^{\circ}C) &= 0.1748 \\ C_S(20^{\circ}C) &= 14.462 \\ \beta &= 0.0590 \end{aligned} \tag{4-5}$$

The temperature dependent parameters are obtained by substituting equation (4-5) into equation (4-1). They are

$$
\begin{aligned}
a_1(\tau) &= 0.1444e^{0.025\tau°C} \\
a_2(\tau) &= 0.1748e^{0.0464\tau°C} \\
C_S(\tau) &= 14.462e^{-0.021\tau°C} \\
D_B(\tau) &= 0.059\ (25 - 0.028\ (\tau°C - 30)^2)
\end{aligned}
\qquad (4\text{-}6)
$$

These temperature dependent parameters are only valid for a temperature range of between 5°C and 35°C.

4.3 Effect of Temperature Variation on Discrete Time Systems

Substituting equation (4-6) for the system parameters in the discrete time models in chapter 2, the initial conditions for each quarter of a year are calculated. These initial conditions are shown in Table 2.

TABLE 2

INITIAL CONDITIONS FOR EACH QUARTER

Start of Each Quarter	$\bar{x}_1(0)$	$\bar{x}_2(0)$	$\bar{U}_A(0)$	$\bar{U}_E(0)$
October First	6.0	6.68	0.31	0.0
January First	6.11	7.31	0.0	0.0
April First	6.20	7.12	0.0	0.0
July First	6.0	6.62	0.70	0.0

The discrete time models are then simulated on a digital computer for a period of 80 days (we have

assumed an input (upstream) DO and BOD profiles for an 80 day period in chapter 3) in each quarter of a year using the discrete time controller designed in chapter 3 and the initial conditions of table 2. The results obtained are presented in figures 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 and 4.8. A discussion of this results will be found in chapter 5.

4.4 Effect of Temperature on the Continuous-Time Systems

After substituting equation 4-6 for the system parameters in the continuous-time model of the river system, the continuous system was also simulated on a digital computer** for a period of 80 days in each quarter of the year. The continuous-time controllers designed by Ramar and Gourishankar [18, 19] are used (see appendix 2). The initial conditions are the same as in table 2. The results obtained are presented in figures 4.9, 4.10, 4.11, 4.12, 4.13, 4.14 and 4.15. A discussion of these results and a comparison of the continuous-time and discrete-time controller performance is given in chapter 5.

** A continuous system modelling program (CSMP) was used in this simulation. This program was obtained from the computing science library.

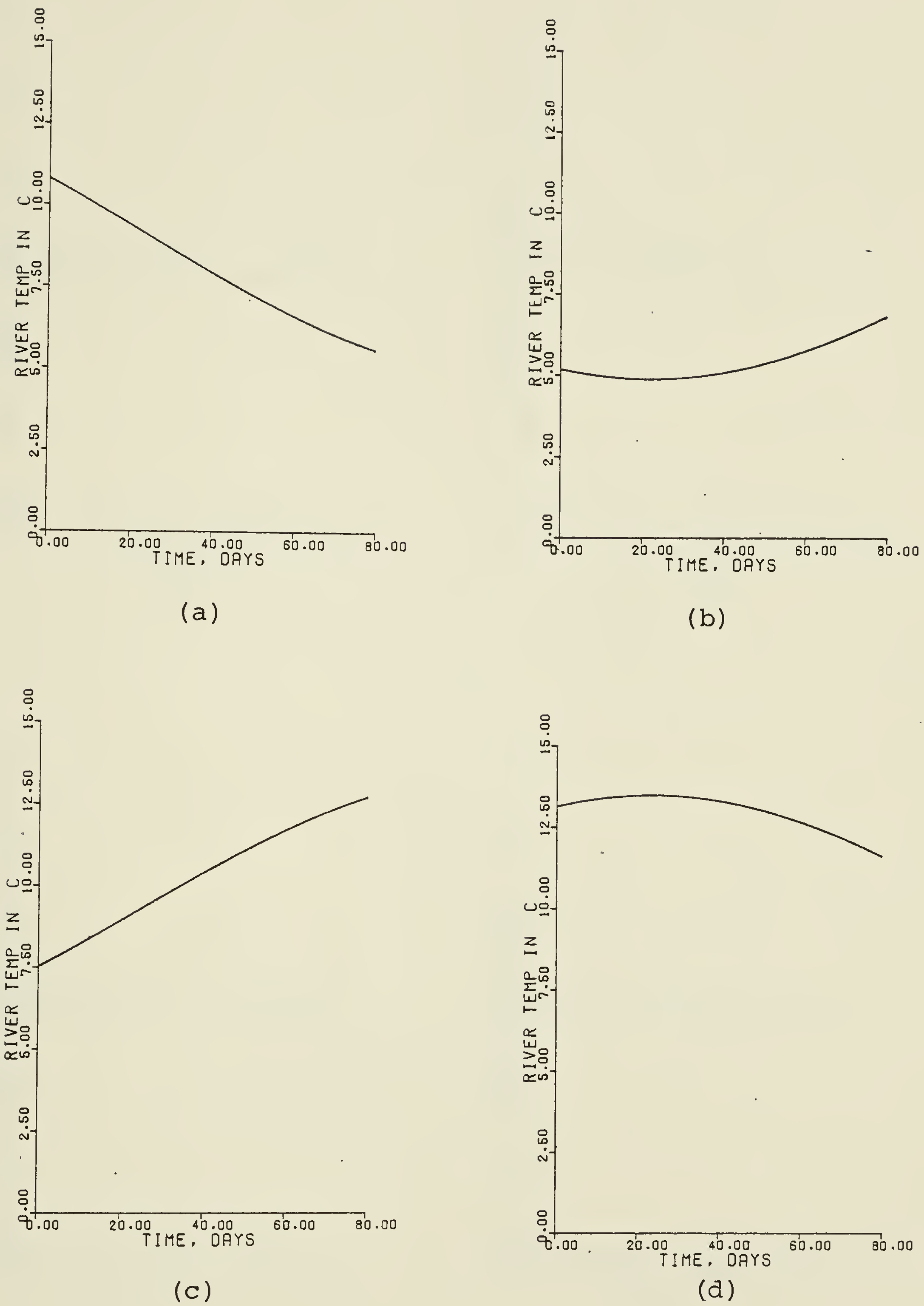


Figure 4.1 Seasonal Temperature Profiles Used for the Investigation of the Effect of Temperature on the Discrete and Continuous-Time Systems

(a) First Quarter (Oct. - Dec.)
(b) Second Quarter (Jan. - March)
(c) Third Quarter (April - June)
(d) Fourth Quarter (July - Sept.)

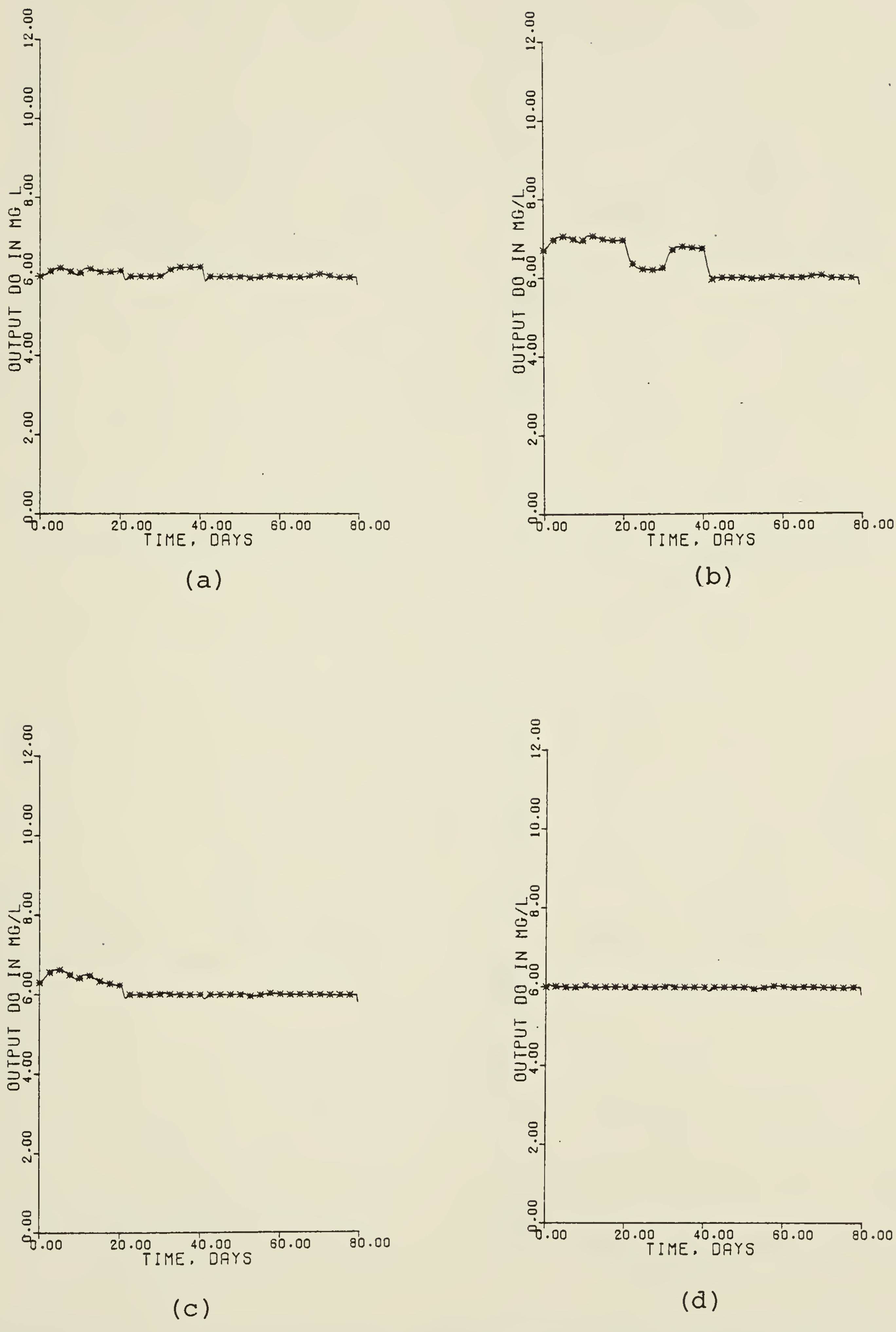


Figure 4.2 Discrete-Time Response (Temperature Dependent Parameters)

(a) Output DO (Oct. - Dec.)
(b) Output DO (Jan. - March)
(c) Output DO (April - June)
(d) Output DO (July - Sept.)

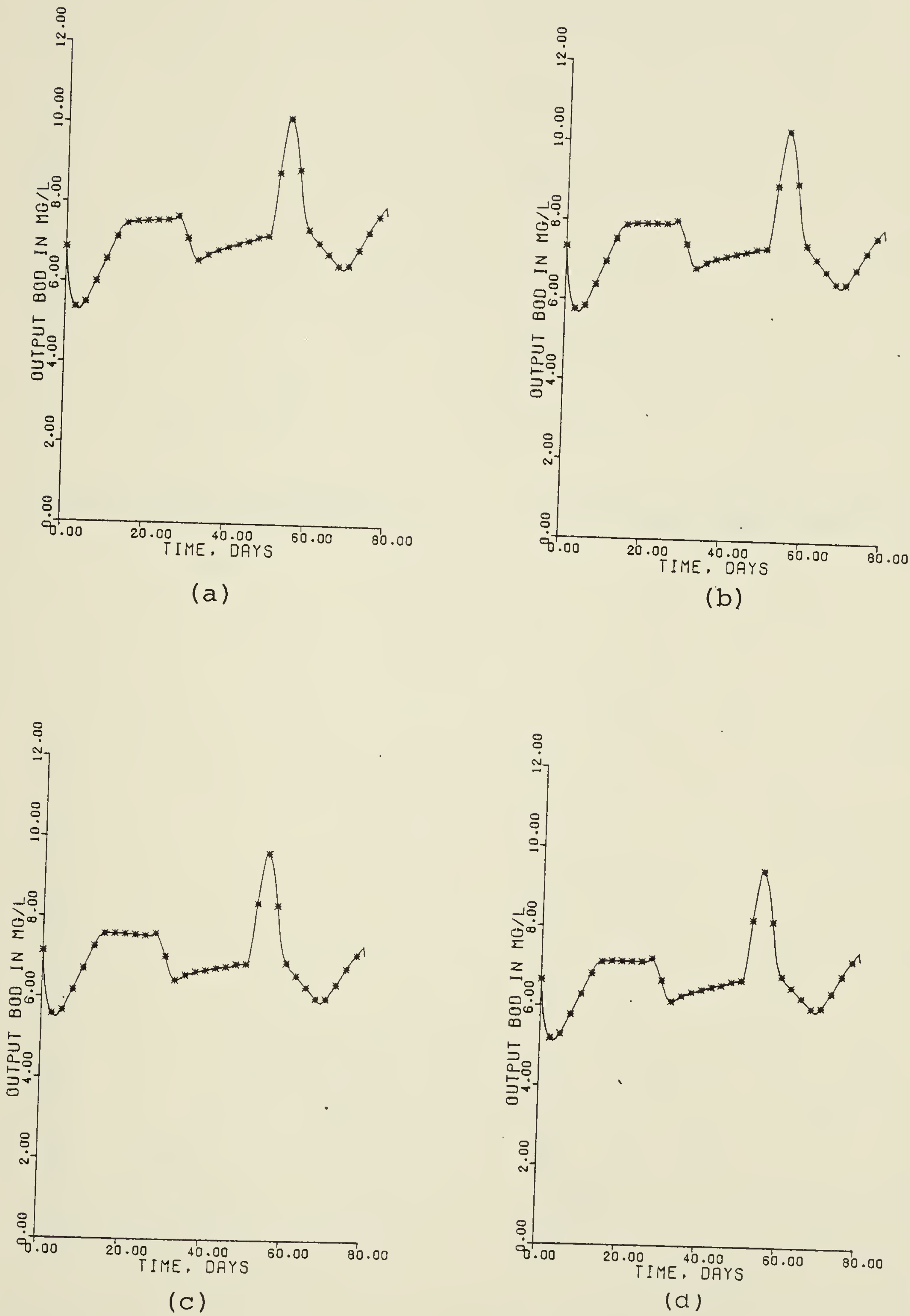


Figure 4.3 Discrete-Time System Response (Temperature Dependent Parameters) Aeration Control Case

(a) Output BOD (Oct. - Dec.) (b) Output BOD (Jan. - March)
(c) Output BOD (April - June) (d) Output BOD (July - Sept.)

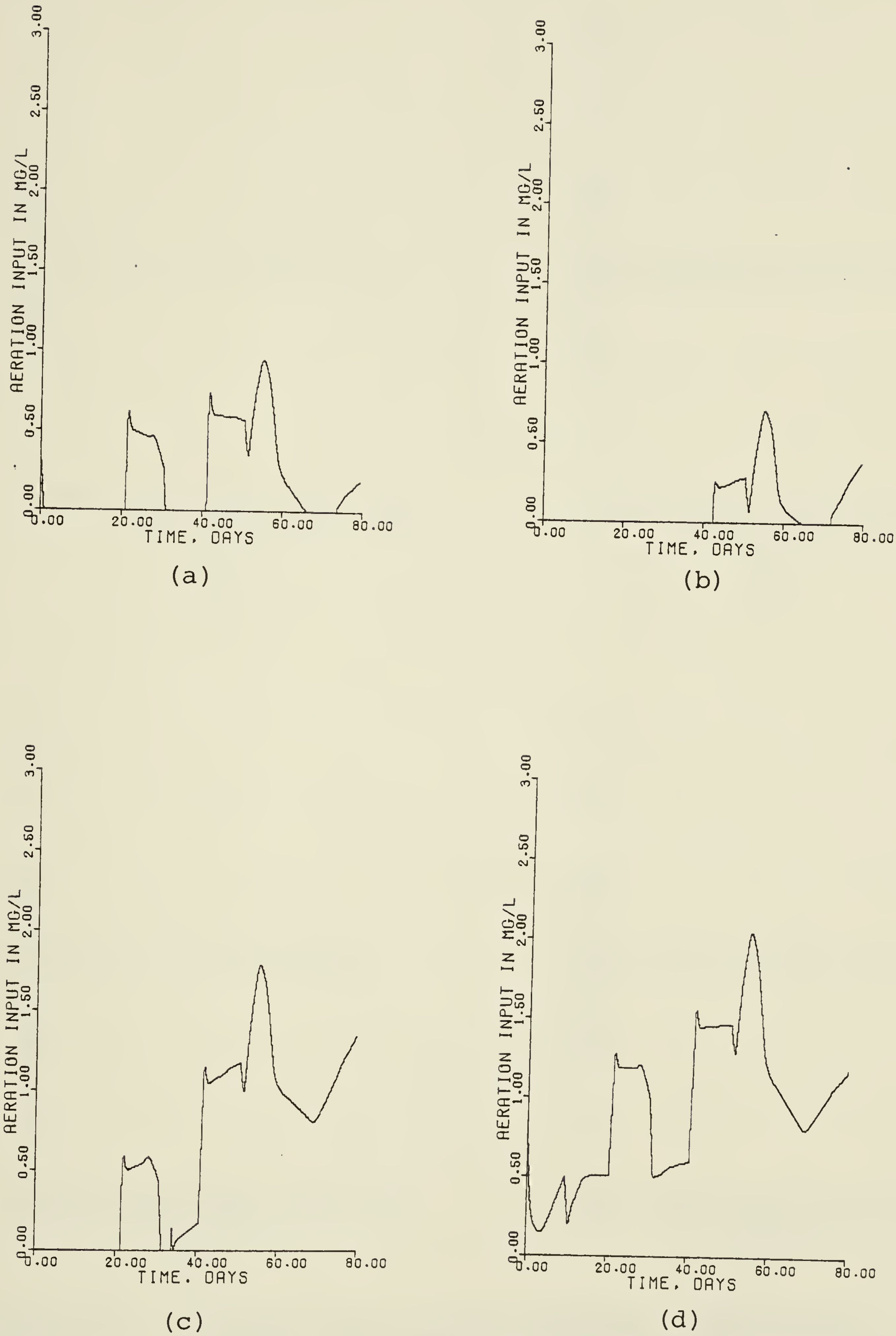


Figure 4.4 Discrete-Time System Response (Temperature Dependent Parameters) Aeration Control Case

(a) Artificial Aeration (Oct. - Dec.)
(b) Artificial Aeration (Jan. - March)
(c) Artificial Aeration (April - June)
(d) Artificial Aeration (July - Sept.)

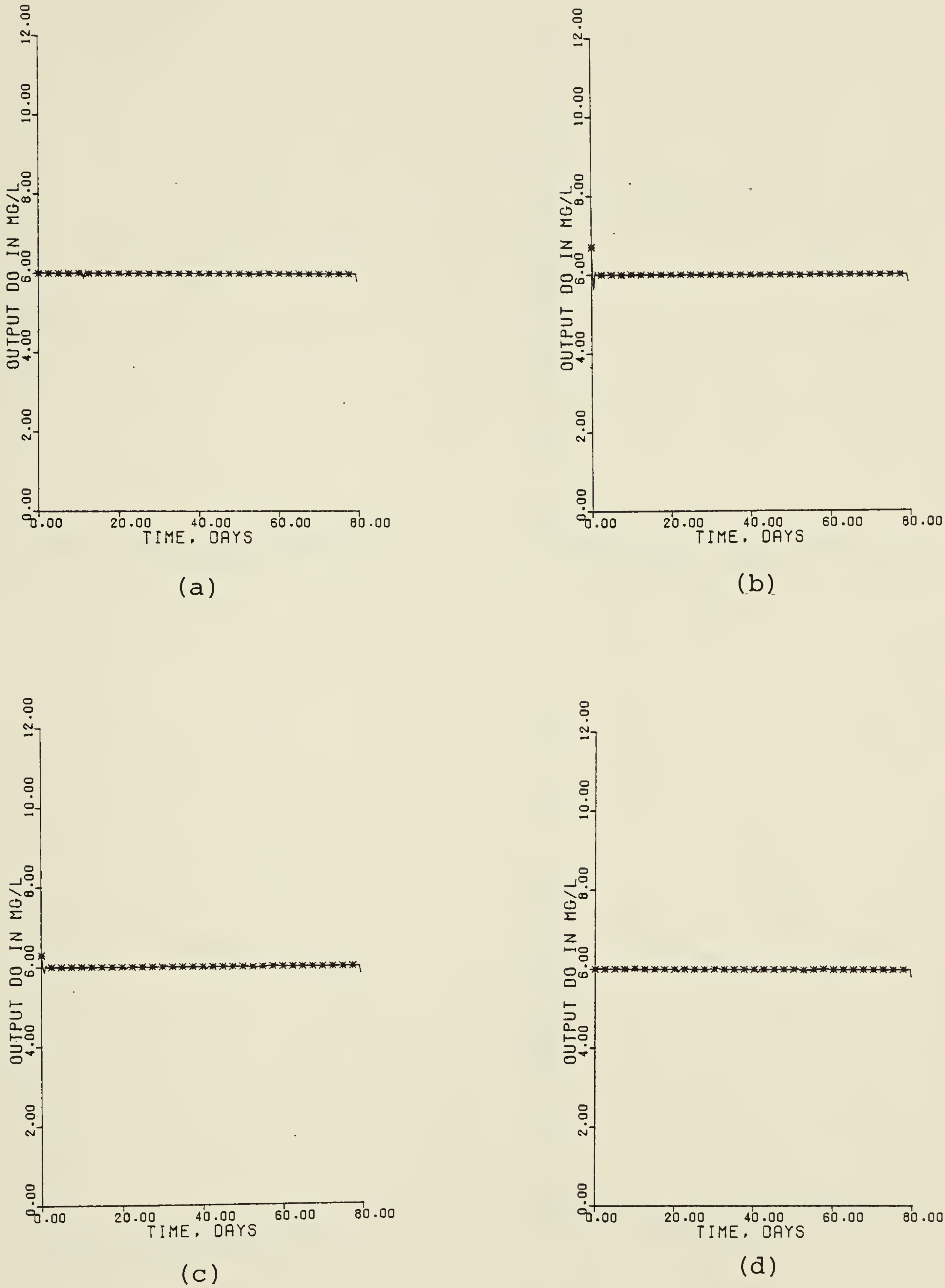


Figure 4.5 Discrete-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Case

(a) Output DO (Oct. - Dec.) (b) Output DO (Jan. - March)
(c) Output DO (Apr. - June) (d) Output DO (July - Sept.)

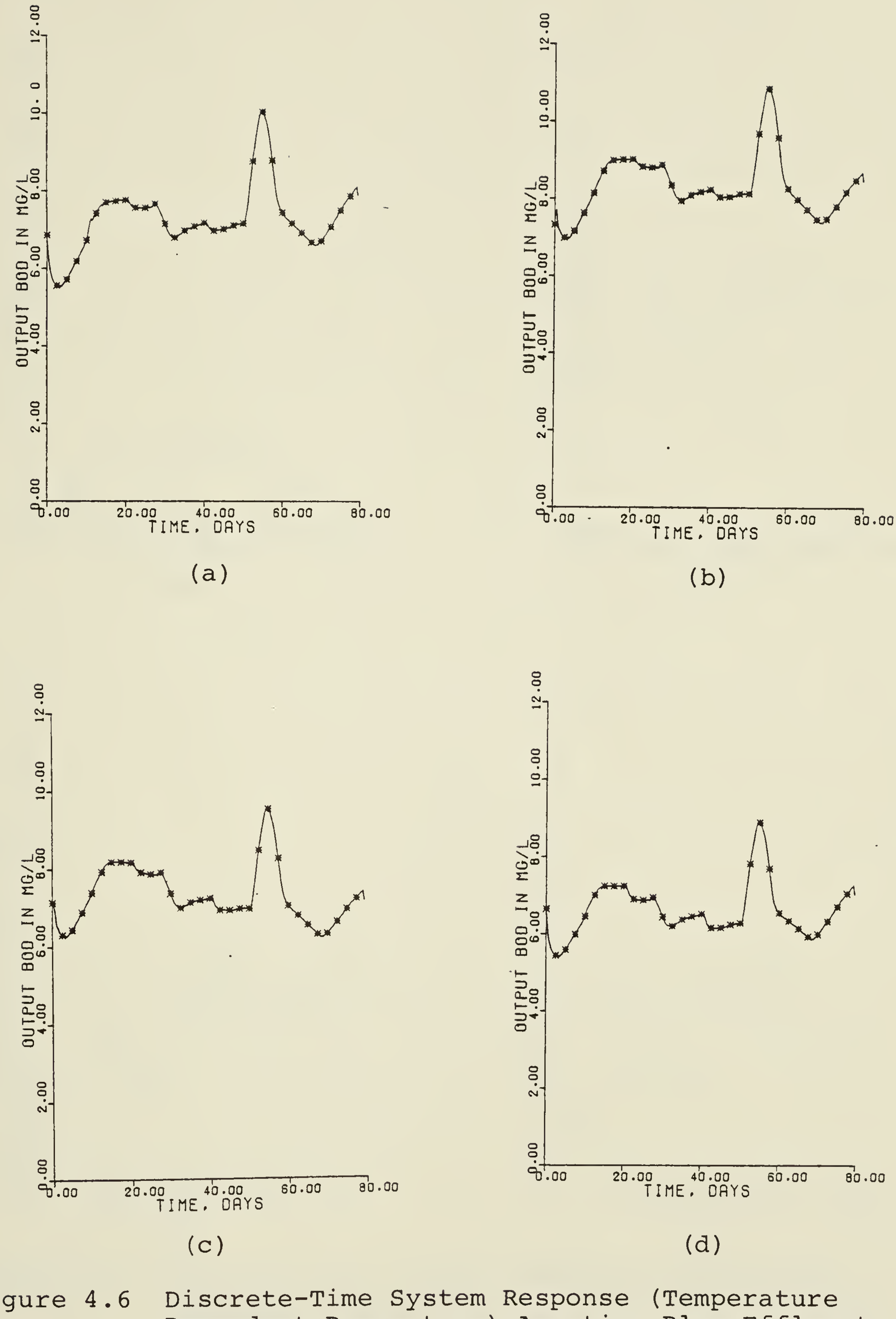


Figure 4.6 Discrete-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Case

(a) Output BOD (Oct. - Dec.) (b) Output BOD (Jan. - Mar.)
(c) Output BOD (Apr. - June) (d) Output BOD (July - Sept.)

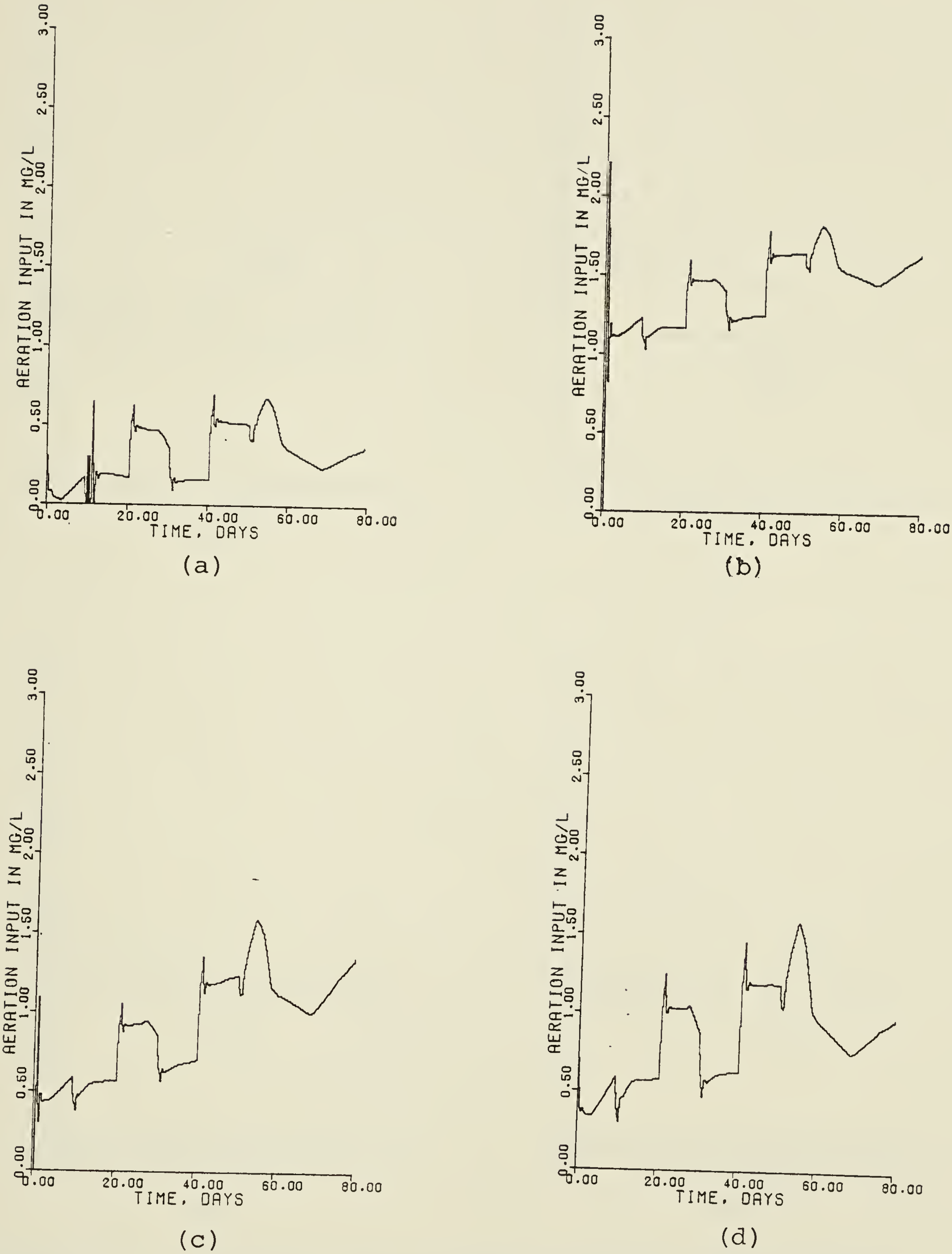
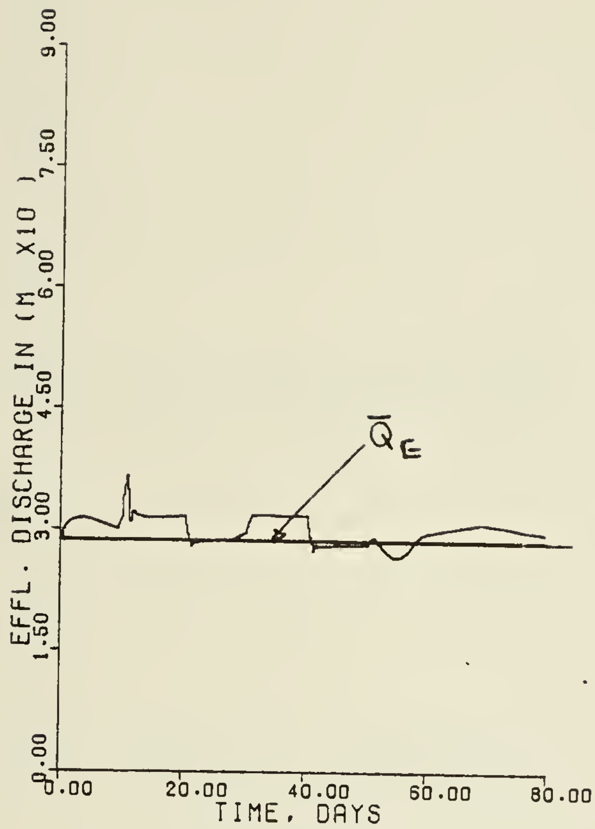


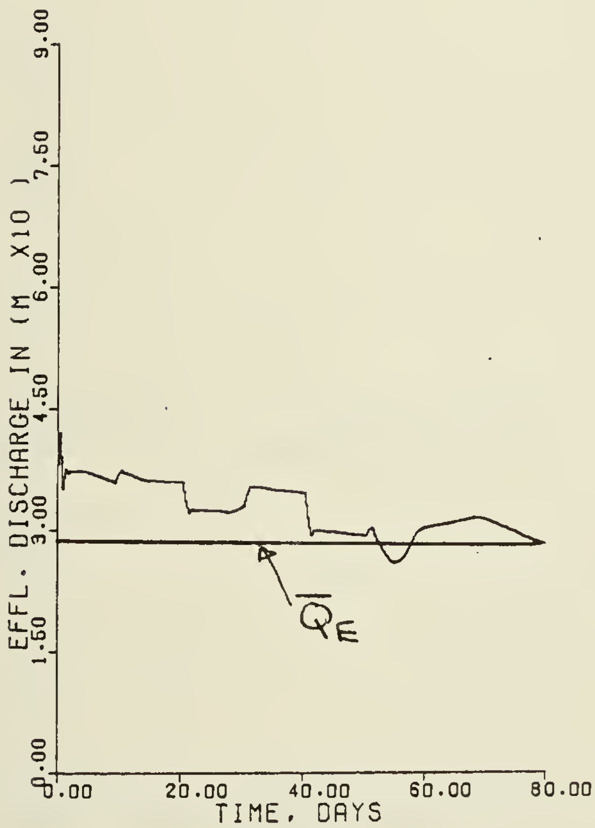
Figure 4.7 Discrete-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Case

(a) Artificial Aeration (Oct. - Dec.)
(b) Artificial Aeration (Jan. - March)
(c) Artificial Aeration (April - June)
(d) Artificial Aeration (July - Sept.)

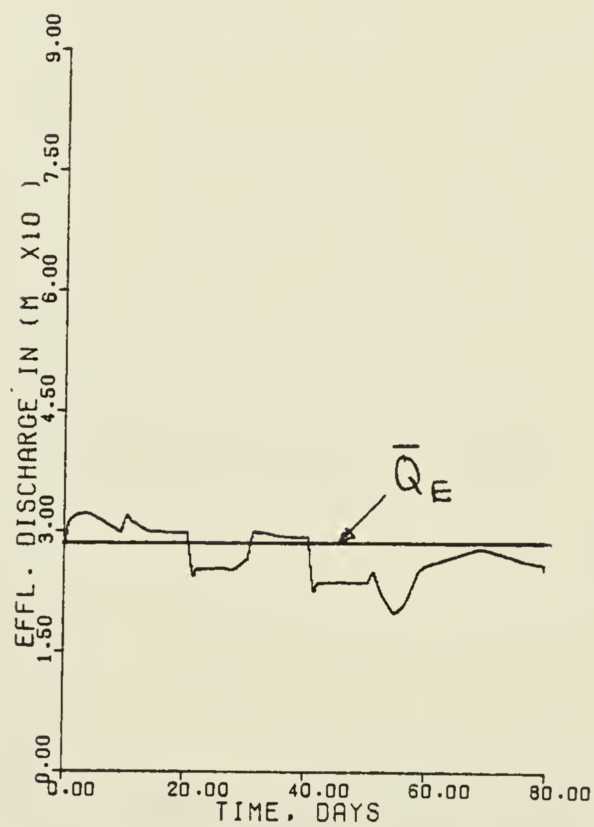


(a)

(b)



(c)



(d)

Figure 4.8 Discrete-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Controls Case

(a) Effluent Discharge (Oct. - Dec.)
(b) Effluent Discharge (Jan. - March)
(c) Effluent Discharge (April - June)
(d) Effluent Discharge (July - Sept.)

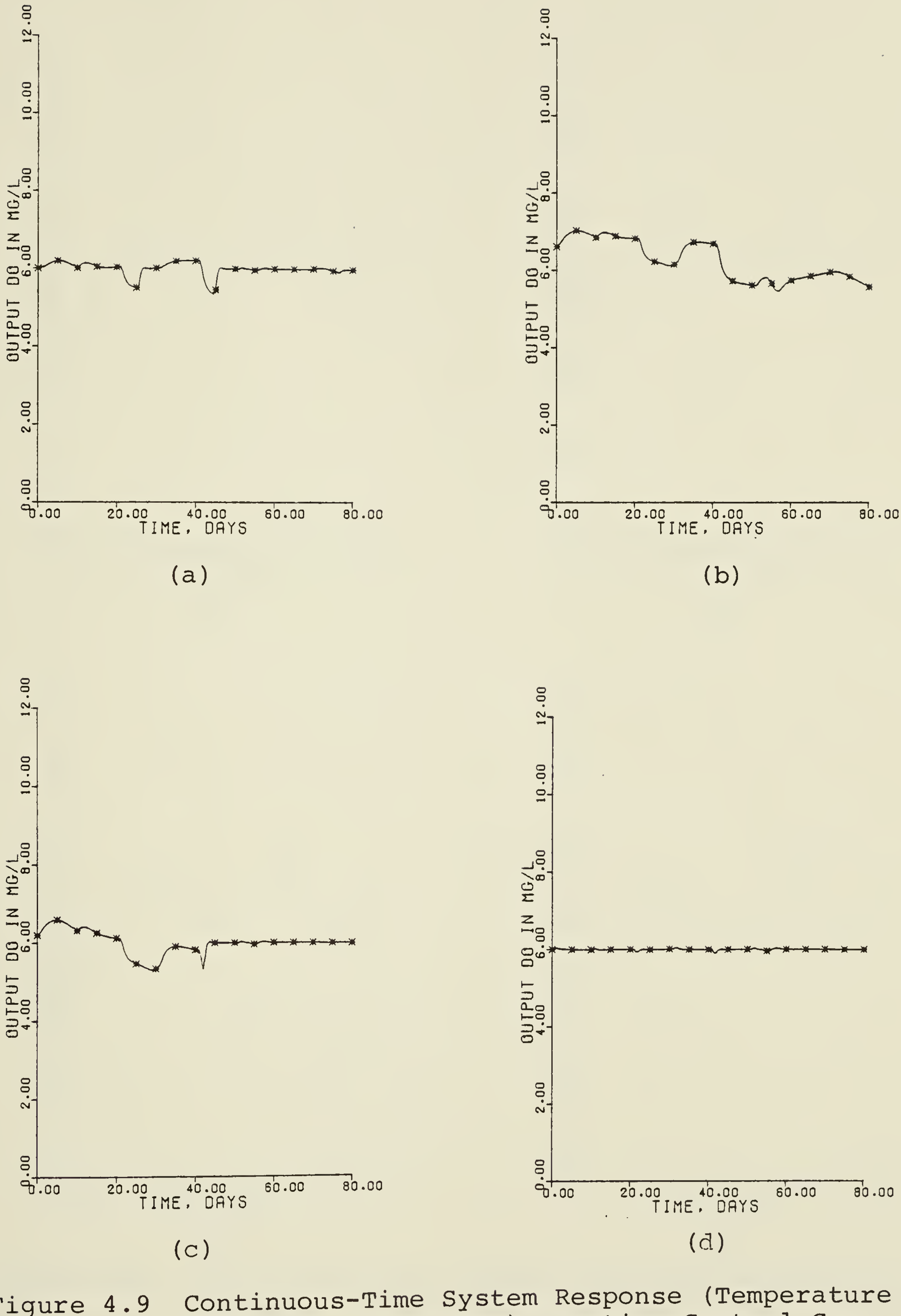


Figure 4.9 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Control Case

(a) Output DO (Oct. - Dec.) (b) Output DO (Jan. - Mar.)
(c) Output DO (Apr. - June) (d) Output DO (July - Sept.)

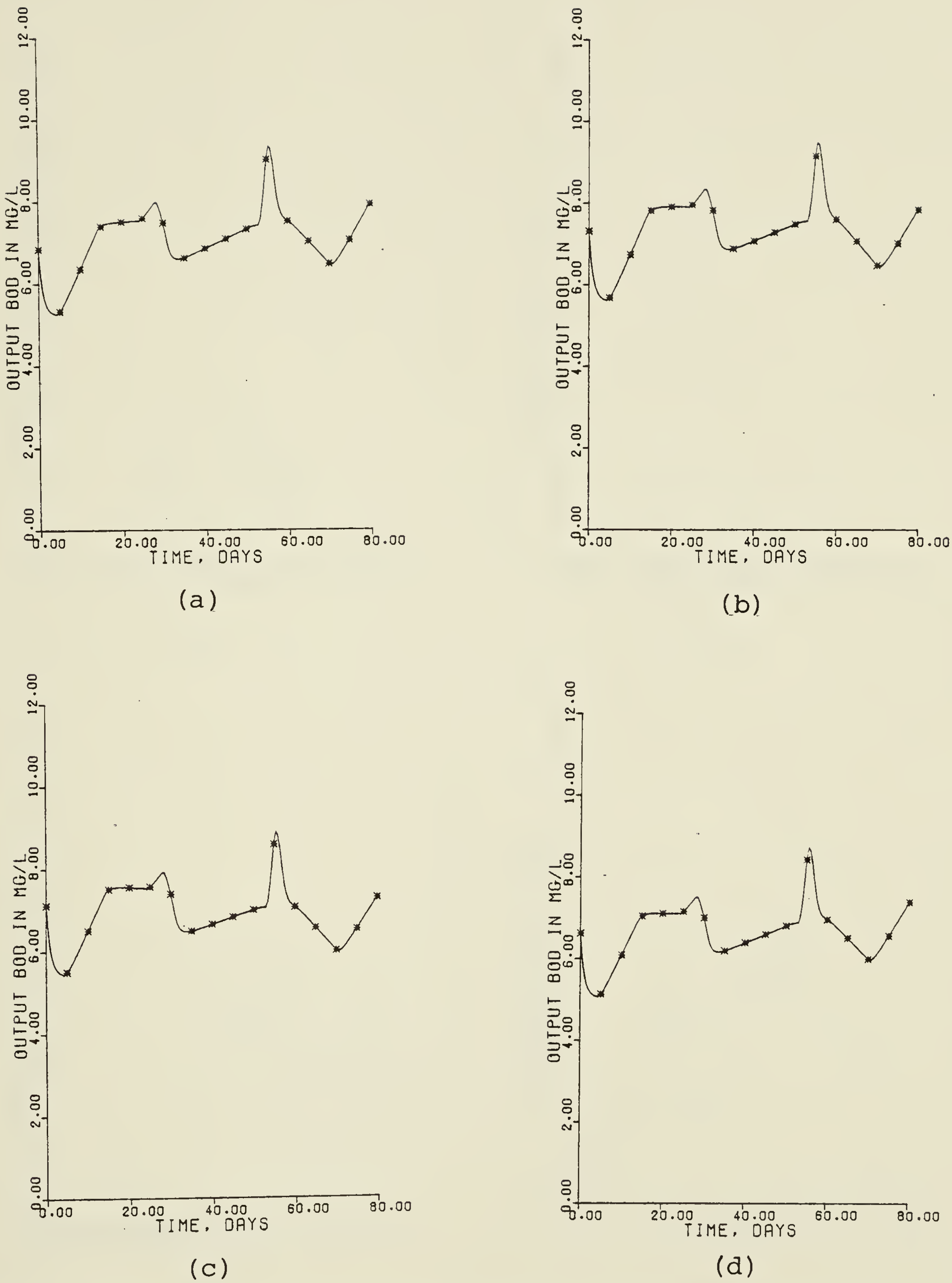


Figure 4.10 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Control Case

(a) Output BOD (Oct. - Dec.)
(b) Output BOD (Jan. - March)
(c) Output BOD (April - June)
(d) Output BOD (July - Sept.)

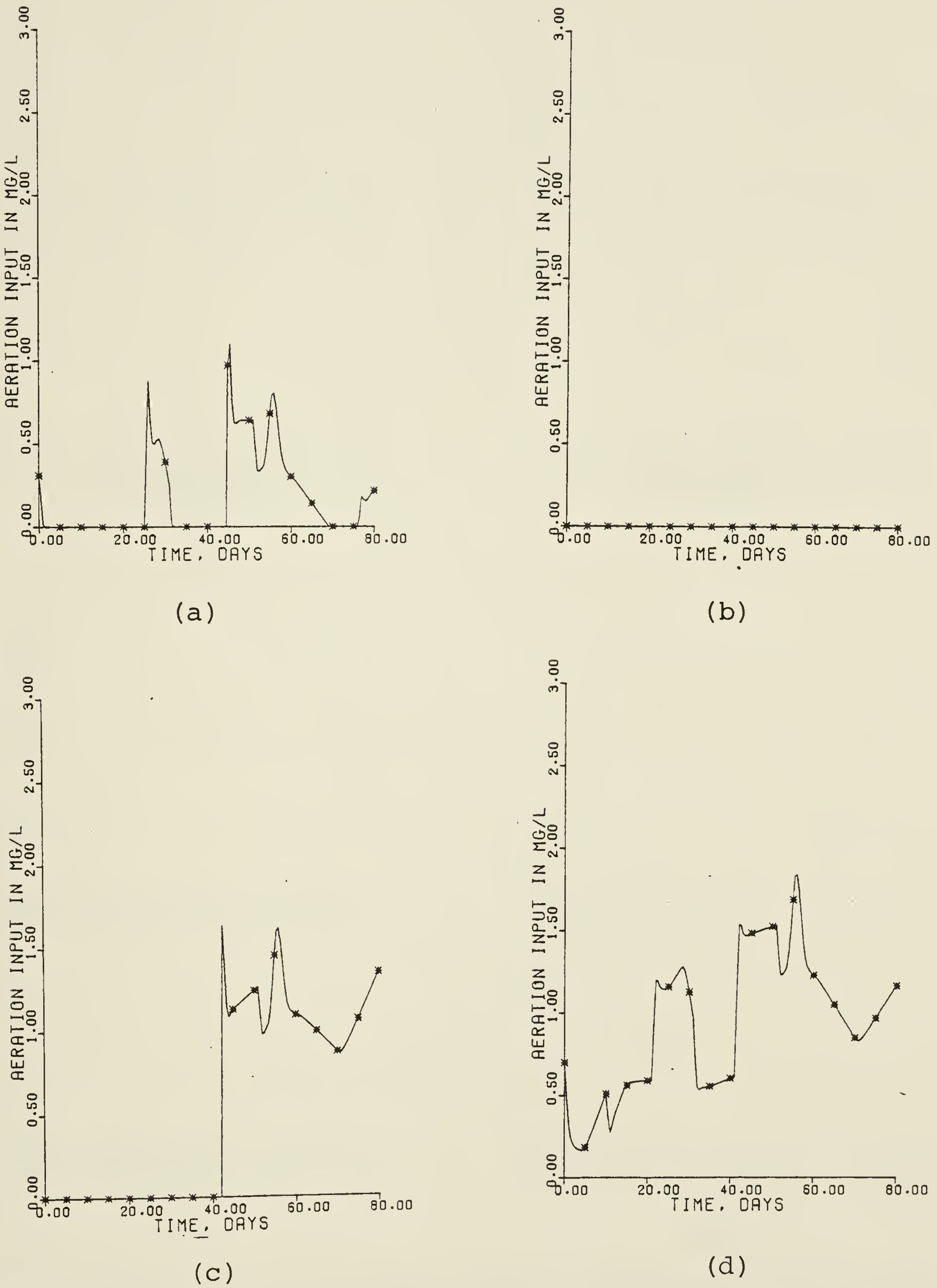


Figure 4.11 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Control Case

(a) Artificial Aeration (Oct. - Dec.)
(b) Artificial Aeration (Jan. - March)
(c) Artificial Aeration (April - June)
(d) Artificial Aeration (July - Sept.)

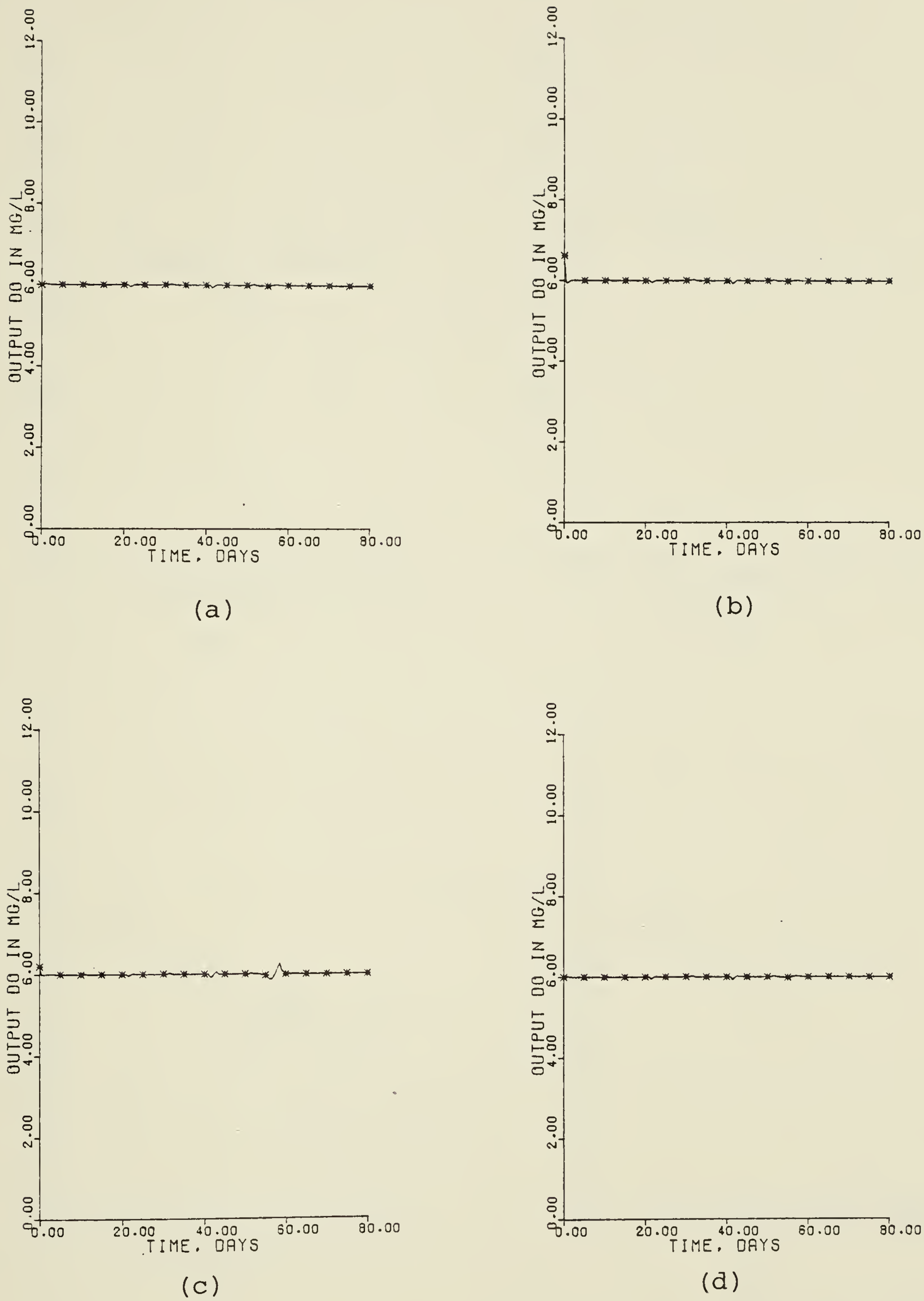
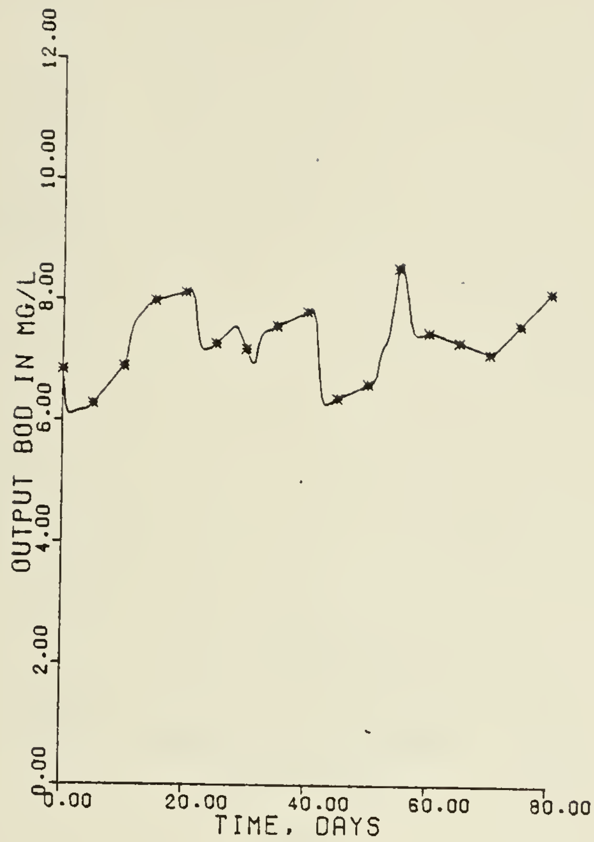
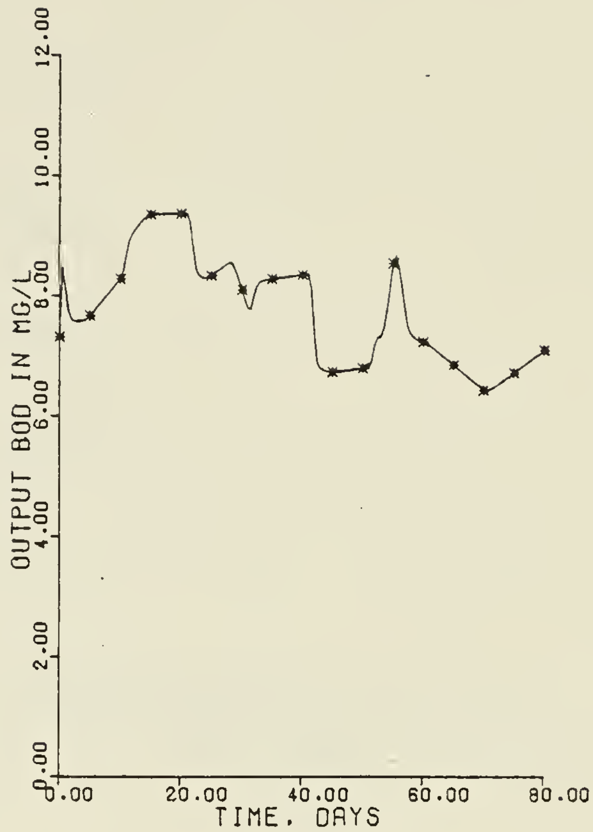


Figure 4.12 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Case

(a) Output DO (Oct. - Dec.) (b) Output DO (Jan. - March)
(c) Output DO (April - June) (d) Output DO (July - Sept.)



(a)



(b)

(c)

(d)

Figure 4.13 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Case

(a) Output BOD (Oct. - Dec.) (b) Output BOD (Jan. - March)
(c) Output BOD (April - June) (d) Output BOD (July - Sept.)

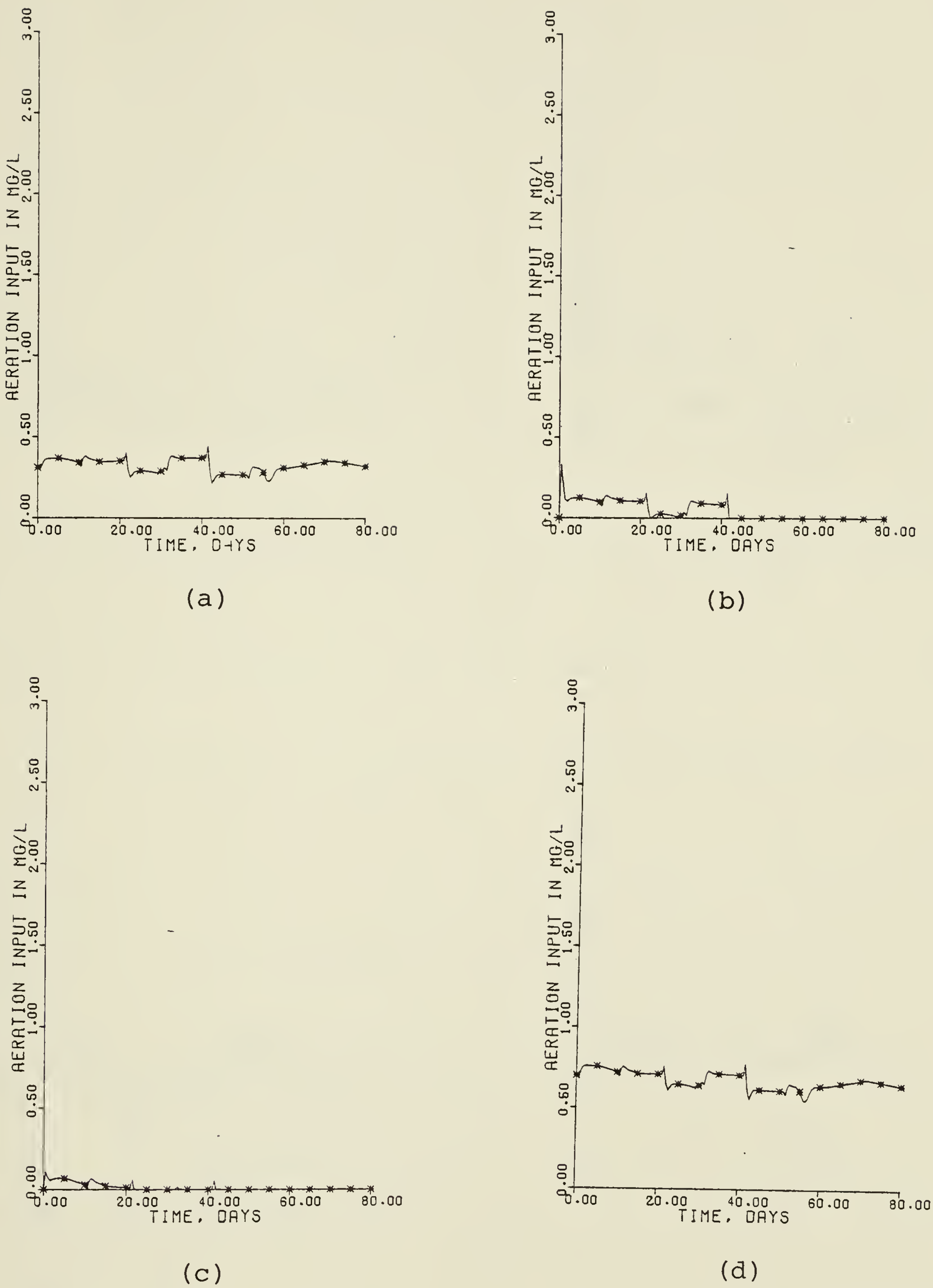


Figure 4.14 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Controls Case

(a) Artificial Aeration (Oct. - Dec.)
(b) Artificial Aeration (Jan. - March)
(c) Artificial Aeration (April - June)
(d) Artificial Aeration (July - Sept.)

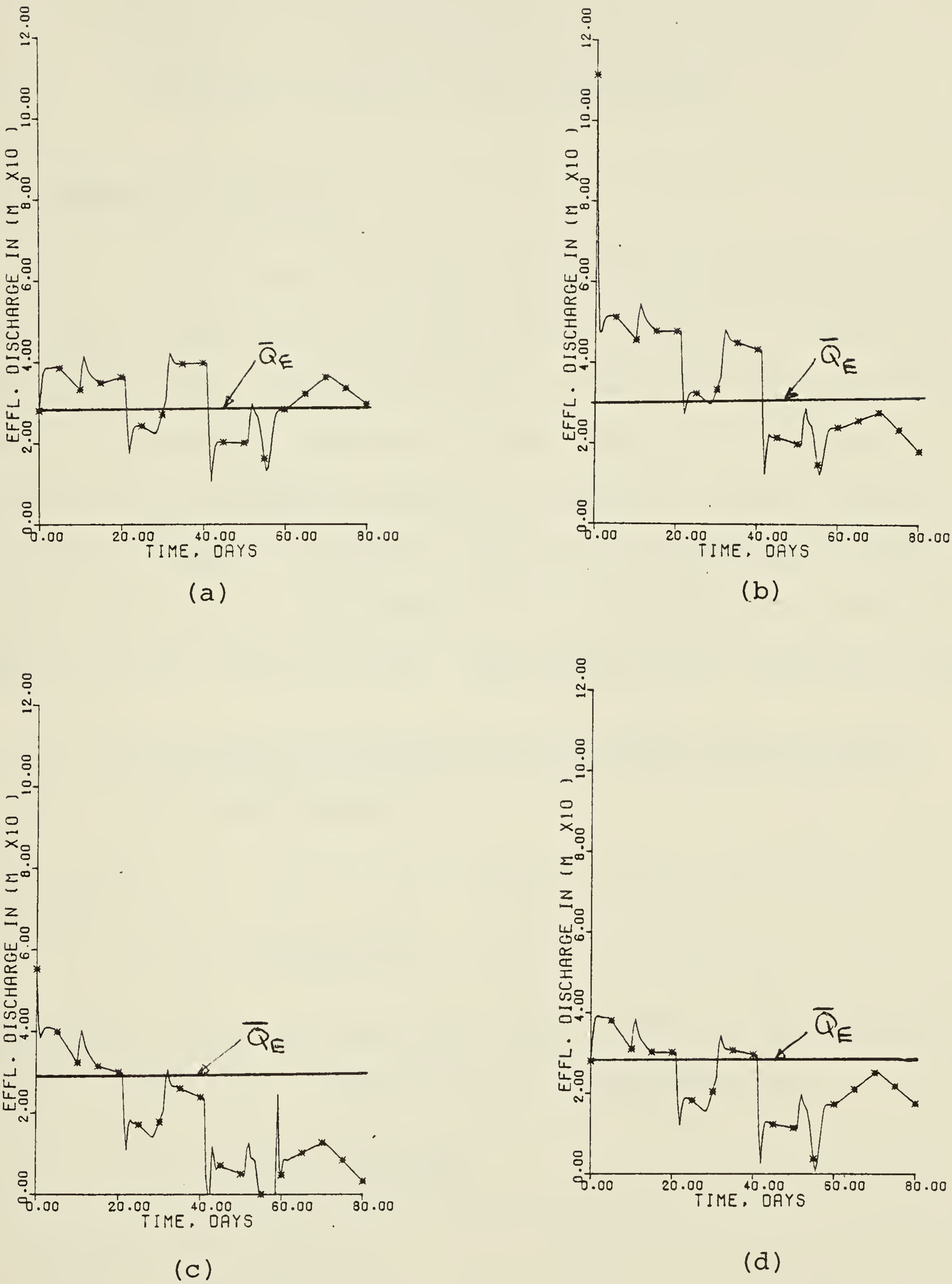


Figure 4.15 Continuous-Time System Response (Temperature Dependent Parameters) Aeration Plus Effluent Discharge Controls Case

(a) Effluent Discharge (Oct. - Dec.)
(b) Effluent Discharge (Jan. - March)
(c) Effluent Discharge (April - June)
(d) Effluent Discharge (July - Sept.)

CHAPTER 5

SUMMARY OF RESULTS AND CONCLUSIONS

5.1 Summary of Results

In the previous two chapters, we derived discrete-time controllers for maintaining the DO level of a polluted river system at a prescribed level. Two cases were considered namely: Artificial aeration control only and Artificial aeration plus effluent discharge controls. We also investigated the effect of seasonal temperature variation on these controllers and the continuous-time controllers designed by Ramar and Gourishankar [18]. The results are summarized below.

1. Discrete-time system with aeration control only

   (a) System model

$$\underline{x}(K+1)T = \Phi(T)\ \underline{x}\ (KT) + B(T)\ \underline{w}\ (KT) + D(T) + E(T)U_A(KT)$$

where $\Phi(T)$, $B(T)$, $D(T)$ and $E(T)$ are defined in equations (2-16), (2-17), (2-18) and (2-19) with $t - t_o$ replaced by T.

   (b) Control function

$$U_A(K) = \begin{cases} Y & \text{for } Y \geq 0 \\ 0 & \text{for } Y \leq 0 \end{cases}$$

where $Y = -2.5224\ E_1(K) + 1.9516E_3(K) + \bar{U}_A$

2. Discrete-time system with aeration and effluent discharge controls

(a) System model

$$\underline{x}(K+1)T = \Phi(K,T)\ \underline{x}\ (KT) + B(K,T)\ \underline{w}\ (KT) + D(K,T) + E(K,T)\ \underline{U}(KT)$$

Where $\Phi(K,T)$, $B(K,T)$, $D(K,T)$ and $E(K,T)$ are defined in equation (2-14).

(b) Controls

$$U_A(K) = \begin{cases} Y_1 & \text{for } Y_1 \geq 0 \\ 0 & \text{for } Y_1 < 0 \end{cases}$$

Where $Y_1 = -3.4462\ E_1(K) + 29110X_3(K) + \overline{U}_A$

$$Q_E(K) = \begin{cases} Y_2 & \text{for } Y_2 \geq 0 \\ 0 & \text{for } Y_2 < 0 \end{cases}$$

where $Y_2 = 15.1 \times 10^4[0.0169E_1(K) - 0.1910X_3(K) + \overline{U}_E]$

3. Effect of seasonal temperature variation

(a) Temperature variation

$$\tau = -4.3[\sin(0.986\gamma - 23.1)] + 9.19°C$$

(b) Temperature dependent parameters

The parameters a_1, a_2, C_s, D_B in the system models are replaced by temperature dependent parameters for the purposes of simulation. These temperature dependent parameters are defined in equation (4-6). In the design of

the controllers however, these parameters were assumed to be constant at their values corresponding to the mean temperature during the summer months (June - August).

5.2 Discussion of Results

(1) One-control scheme ie. aeration control only - - Effluent discharge kept constant

(a) Constant parameters

The results obtained for the aeration control case with constant parameters are shown in figure 3-4. These results are similar to those obtained by Ramar and Gourishankar [18] for the continuous controller.

(b) Effect of temperature variation on the discrete controller

The results obtained are shown in figures 4.2, 4.3 and 4.4. The results show that the amount of artificial aeration required to maintain the desired level of DO at 6 mg/ℓ decreases as the temperature of the river decreases. This is due to the increase in the saturation level of the dissolved oxygen in the river and the increase in the difference between the reaeration rate and the BOD decay rate. These two factors produce a net increase in the amount of oxygen available for the decomposition of the waste. Since in this case the effluent discharge is kept constant, the increase in the amount of oxygen available results in a decrease in the amount of artificial aeration required.

(c) Effect of temperature variation on the continuous controler

The results obtained for the continuous controller are shown in figures 4.9, 4.10, and 4.11. These results show unsatisfactory DO response, especially for the first three quarters (Oct. - June) of the year. This is because the controller was not turned off when the DO level is higher than the desired level. A non-linear control function will be required in order to improve the performance of this controller.

(d) Comparison of continuous-time controller and the discrete time controller

As mentioned in section (a) above the results obtained for the discrete-time controller with constant parameters compare very well with the continuous-time controller. The number of observations required to maintain the output DO level of the river had been reduced to twice per day without any sacrifice in the performance of the system. The system could easily be implemented by a human operator using a set of control valves. Since the effluent discharge is kept constant in this scheme, it might be necessary to store the effluent in a lagoon so as to smooth out the variation in the production of effluent from the sewage plant and its dumping into the river.

2. Two-control scheme ie. aeration plus effluent discharge

(a) Constant Parameters

The results obtained for the two controls case with constant parameters are shown in figure 3.7. These

results also compare very well with those obtained by Ramar and Gourishankar [19]. While the volume of effluent discharged for the continuous system varies from about 40,000 to 0 cubic meters with a lot of fluctuations, the volume of effluent discharge for the discrete time systems never goes below 20,000 cubic meters and the fluctuations are a lot less. However, the amount of artificial aeration required to maintain the desired level of DO in the river is more than that required in the continuous case.

(b) Effect of temperature variation on the discrete time controller

The results are shown in figures 4.5, 4.6, 4.7 and 4.8. These results show that the amount of effluent discharged into the river increases as the temperature of the river decreases. Thus more waste will be assimilated by the river during the winter months. The reasons are as stated in section 5.2 subsection 1(b) of this chapter. This increase in the amount of effluent however, is also accompanied by an increase in the amount of artificial aeration required to maintain the desired level of DO in the river.

(c) Effect of temperature variations on the continuous-time controller

The results for this part of the investigation are shown in figures 4.12, 4.13, 4.14 and 4.15. The results show the same trend as for the discrete time controller ie. the amount of effluent discharged increases as temperature decreases, however, this increase in effluent discharge is followed by a decrease in the amount of artificial aeration

required to maintain the level of DO at 6 mg/ℓ. This is different from what was obtained for the discrete time controller. This might be due to the fact that the sensitivity of the closed loop pole to parameter variation was minimized in the design of the continuous controller, this was not done in the design of the discrete time controller, instead the minimization of the sensitivity of the output DO to parameter variation was used in determining the gain of the discrete-time controller.

(d) Comparison of the discrete-time system and the continuous time system

The results show the same trends as far as effluent discharge is concerned. The amount of artificial aeration required however, show opposite trends, but it should also be noted that the fluctuations in the amount of effluent discharged is greatly reduced for the discrete time system. The amount of effluent discharged is more during the first half of the 80 day period than the second half for the continuous system.

5.3 Conclusion

Apart from the increase in the amount of artificial aeration required, it had been shown in this discussion that the performance of the discrete-time system compares very well with that of the continuous system. With the reduced number of observations required, two per day for the constant parameter case, and four per day for the temperature dependent

parameter case, the implementation of this control schemes are much simpler than the implementation of the continuous control scheme. As stated in section 5.2 subsection 1(d) a lagoon might be required for temporary storage of the effluent.

5.4 Suggestion for Further Research

The application of automatic control theory to environmental problems is still in the development stage. The discussion in this thesis is therefore not intended to offer a solution to the problem of organic waste pollution in our river systems. What has been attempted is to outline a procedure for obtaining a solution to the problem using digital control techniques. The following are areas for further studies.

(1) In chapter two, digital models were derived from a lumped parameter continuous model, since the validity of the parameters of these models is restricted to a limited temperature range, a study could be conducted to determine the relationships between these parameters and temperature for lower temperatures up to and including the freezing point of the river, followed by the use of the pole assignment technique to regulate the output variable of the system model.

(2) The investigation carried out in chapter 4 had assumed that the river is in thermal equilibrium with the atmosphere. In general, thermal equilibrium is never reached.

This investigation could therefore be extended to include the heat exchange mechanism between the river and the atmosphere. Also the effect of other heat sources, such as dumping of water from the cooling towers of power generating plants into the river could be investigated.

BIBLIOGRAPHY

1. CAMP, T.R. Water and its impurities. Reinhold Publishing Co., 1963.

2. COMMITTEE ON SANITARY ENGINEERING RESEARCH. "31st progress report on the effect of water temperature on stream reaeration". Journal of Sanitary Engineering SA(6), November 1961.

3. DAVIDSON AND BRADSHAW. "Thermal Pollution of Water Systems". Environmental Science and Technology. Vol. 1, No. 8, August 1967.

4. DEININGER, R.A. "Models for Environmental Pollution Control". Ann Arbor Science Club, 1973.

5. DI TORO, D.M. "Stream equation and method of characteristics". American Society of Civil Engineers Proceedings, SA(3) 95, August 1969.

6. DOBBINS, W.E. "BOD and oxygen relationships in streams". Journal of Sanitary Engineering, American Society of Civil Engineers 90, SA(3), 1964.

7. ECKENFELDER, W.W. JR. "Water quality engineering for practicing engineers". New York, Barnes and Noble, 1970.

8. GOURISHANKAR, V., LAWSON, R. "Optimal control of water pollution in river systems". International Journal of Systems Science, Vol. 6, No. 3 (1975).

9. HELFRICH, H.W. JR. "Environmental crisis". Yale University Press, 1970.

10. HOROWITZ, M. "The sensitivity problem in sample data systems". Institute of Radio Engineers Transcript on Automatic Control, Vol. 6, pp. 251, 1961.

11. KUDVA, P., GOURISHANKAR, V. "Observer based water quality controller for polluted river streams". International Journal of Systems Science (to appear).

12. KUO, B.C. Discrete data control systems". Science Technology, 1974.

13. LINDORFF, D.P. "Sensitivity in sampled data systems". Institute of Radio Engineers Transactions on Automatic Control, Vol. 8, April, 1963.

14. NEMEROW, N.L. "Scientific stream pollution analysis". Scripta Book Company, 1974.

15. O'CONNOR, D. "The temporal and spartial distribution of DO in streams". Water Resources Research, 3, Vol. 1, 1967.

16. O'CONNOR, D., DOBBINS, W. "The mechanism of reaeration in natural streams". Journal of Sanitary Engineering, American Society of Civil Engineers SA(6), Dec., 1956.

17. PARKER, F.L., KRENKEL, P.A. "Physical and engineering aspects of thermal pollution". Vanderbilt University, CRC Press.

18. RAMAR, K., GOURISHANKAR, V. "Control of dissolved oxygen concentration in polluted streams by feed back control techniques". International Journal of Systems Science. (to appear).

19. GOURISHANKAR, V., RAMAR, K. "Design of a water quality controller by pole assignment". Institute of Electronic and Electrical Engineers Transactions on Systems, Man and Cybernetics, Feb. 1977.

20. SOLIMAN, J.A., KEVORKIAN, A.K. "Parameter variations in multivariable sample data systems". Article D'International Federation of Automatic Control Symposium on System Sensitivity and Adaptivity. Dubrovnik Yugoslavia, August 26-31st, 1968.

21. STREETER, H.W., PHELPS, E.B. "A study of the purification of the Ohio river". U.S. Public Health Service Bulletin No. 146, 1925.

22. TARASSOV, PERLIS, AND DAVIDSON, B. "Optimization of a class of river aeration problems by the use of multi-variable distributed parameter control theory". Water Resources Research Vol. 5, No. 3, March 1969.

23. WARD, J.C. "Annual variation of stream water temperature". Journal of Sanitary Engineering, American Society of Civil Engineers, Dec. 1963.

24. WIBERG, M.D. "State space and linear systems". Schaum's Outline Series.

25. YOUNG AND BECK. "The modelling and control of water quality in a river stream". Automatica 10, 1974.

APPENDIX 1

DERIVATION OF LEAST SQUARE FIT FOR A SINE CURVE

$$t = a \sin(bx + c) + \bar{t}$$

$$= p \sin bx + q \cos bx + \bar{t}$$

where

$$c = \tan^{-1}(q/p)$$

$$a = \sqrt{p^2 + q^2}$$

The deviation of t from observed data t_d

$$= t - t_d$$

$$= p \sin bx + q \cos bx + \bar{t} - t_d = R$$

The sum of the square of this deviation should be minimum for least square fit.

Thus

$$\Sigma R^2 = \Sigma [p \sin bx + q \cos bx + \bar{t} - t_d]^2 = \text{minimum}$$

differentiating w.r.t. P, q and $\bar{t}$ and setting the derivatives to zero gives

$$2\Sigma [p \sin bx + q \cos bx + \bar{t} - t_d] \sin bx = 0$$

$$2\Sigma [p \sin bx + q \cos bx + \bar{t} - t_d] \cos bx = 0$$

$$2\Sigma [p \sin bx + q \cos bx + \bar{t} - t_d] = 0$$

This is equal to

$$P\Sigma \sin^2 bx + q \Sigma \sin bx \cos bx + \bar{t} \Sigma \sin bx - \Sigma t_d \sin bx = 0$$

$$p \, \Sigma \sin bx \;\; \text{cox} \, bx + q \, \Sigma \cos^2 bx + \bar{t} \, \Sigma \cos bx - \Sigma \, t_d \cos bx = 0$$

$$p \, \Sigma \sin bx + q \, \Sigma \cos bx + \Sigma\bar{t} - \Sigma t_d = 0$$

from m pairs of observed data for x and t_d the last equation can be written as

$$p \, \Sigma \sin bx + q \, \Sigma \cos bx + m\bar{t} - \Sigma t_d = 0$$

Therefore solving the following equations simultaneously gives p, q and $\bar{t}$.

$$p \, \Sigma \sin^2 bx + q \, \Sigma \sin bx \cos bx + \bar{t} \, \Sigma \sin bx - \Sigma \, t_d \sin bx = 0$$

$$p \, \Sigma \sin bx \cos bx + q \, \Sigma \cos^2 bx + \bar{t} \, \Sigma \cos bx - \Sigma \, t_d \cos bx = 0$$

$$p \, \Sigma \sin bx + q \, \Sigma \cos bx + m\bar{t} - \Sigma \, t_d = 0$$

APPENDIX 2

CONTROL FUNCTIONS USED IN THE SIMULATION OF THE CONTINUOUS SYSTEMS

Aeration Control Only

$$U_A(t) = -3.26\, E_1(t) + 8.82\, E_3(t) + \bar{U}_A$$

Aeration Plus Effluent Discharge Controls Case

$$U_A(t) = -1.59\, E_1(t) - 2.76\, E_3(t) + \bar{U}_A$$

$$U_E(t) = 13.66\, E_1(t) - 52.11\, E_3(t) + \bar{U}_E$$

The controls are the same as those obtained by Gourishankar and Ramar [18,19].

APPENDIX 3

COMPUTER PROGRAMS USED FOR THE SIMULATION OF CLOSED LOOP SYSTEMS

1. WPC (one-control, constant parameters)
2. WPC2 (2-control, constant parameters)
3. SEW1 (one-control, temperature dependent parameters)
4. SEW (2-controls, temperature dependent parameters)
5. WPT (continuous system, one-control, temperature dependent parameters)
6. WPD (continuous system, two-controls, temperature dependent parameters)

All programs are written in Fortran. WPT and WPD use the continuous systems modelling program (CSMP) available in the computing science program library. All programs were run on the Amdhal 470 V6 Computer of the University of Alberta Computing Services Department.

A listing of these programs is included in this Appendix.

```
LIST  WFC
      C   PROGRAM TO SIMULATE CLOSED LOOP RIVER SYSTEM(CONSTANT PARAMETERS)
            REAL X1(162), X2(162),DO(162),BOD(162),UA(162),TDO(80),TBOD(80)
            REAL TIM(162)
            REAL TIM(162)
            T=0.5
            TIM(1)=0.0
            TIM(1)=0.0
            TIM(161)=0.0
            TIM(162)=20.0
            X1(161)=0.0
            X1(162)=2.0
            X2(161)=0.0
            X2(162)=2.0
            UA(161)=0.0
            UA(162)=0.5
            DO(161)=0.0
            DO(162)=1.0
            BOD(161)=0.0
            BOD(162)=2.0
            TIM(161)=0.0
            TIM(162)=20.0
            X1(161)=0.0
            X1(162)=2.0
            X2(161)=0.0
            X2(162)=2.0
            UA(161)=0.0
            UA(162)=0.5
            DO(161)=0.0
            DO(162)=2.0
            BOD(161)=0.0
            BOD(162)=2.0
            DO 30 K=1,80
         30 READ(5,5000) TDO(K),TBOD(K)
            DO 40 K=1,80
            DO(2*K-1)=TDO(K)
            DO(2*K)=TDO(K)+(TDO(K+1)-TDO(K))/2
            BOD(2*K-1)=TBOD(K)
         40 BOD(2*K)=TBOD(K)+(TBOD(K+1)-TBOD(K))/2
       5000 FORMAT(2F4.1)
            XX1=-0.9417*T
            XX2=-1.0617*T
            X1(1)=6
            UA(160)=9.0
            XL=1
            A=EXP(XX1)
            TIM(K+1)=TIM(K)+0.5
            C=EXP(XX2)
            B=-2.67*(A-C)
            D=0.6*(1-A)
            E=-0.18*(1-8.85*A+7.85*C)
            F=0.53*(1-C)
            G=1.67*(1-A)-1.187*(1-8.85*A+7.85*C)
            H=3.5*(1-C)
            AA=(1-A)/0.9417
            X2(1)=(F*6+H)/(1-C)
            U1=(X1(1)*(1-A)-B*X2(1)-D*10.0-E*6-G)/AA
            UA(1)=U1
            DO 50 K=1,159
            TIM(K+1)=TIM(K)+0.5
            UA(K+1)=UA(K)
            ER=X1(K)-X1(1)
            IF(ER.GT.0) GO TO 78
         49 X3=X3-ER
            UA(K)=-2.522*ER+1.951*X3+U1
            IF(UA(K).LT.0) UA(K)=0
         88 GO TO 66
         78 IF(UA(K).EQ.0) GO TO 88
            GO TO 49
         66 X2(K+1)=C*X2(K)+F*BOD(K)+H
         50 X1(K+1)=A*X1(K)+B*X2(K)+D*DO(K)+E*BOD(K)+G+AA*UA(K)
            CALL XPLOT(TIM,X1,X2,UA,X1,DO,BOD,0,1)
            WRITE(6,6012)
       6012 FORMAT('INPUT AERATION CONTROL')
            WRITE(6,200) UA
            WRITE(6,200) (X1(K),K=1,160)
            WRITE(6,6010)
       6010 FORMAT('OUTPUT DO MEASURED TWICE DAILY')
            WRITE(6,200) (X1(K),K=1,160)
        200 FORMAT(20(2X,F4.2))
            WRITE(6,6011)
       6011 FORMAT('OUTPUT BOD MEASURED TWICE DAILY')
            WRITE(6,200) (X2(K),K=1,160)
            WRITE(6,6013)
       6013 FORMAT('SYSTEM PARAMETERS')
            WRITE(6,200) A,B,C,D,E,F,G,H
            STOP
            END
#END OF FILE
#
```

```
LIST WFC2
      C PROGRAM TO SIMULATE CLOSED LOOP RIVER SYSTEM (2-CONTROL, CONSTANT PARAMETERS)
            REAL X2(162), X1(162),DO(160),BOD(160),UA1(162),UA2(162),TDO(80),TBOD(30)
            REAL TBOD(80),TIM(162)
            T=0.5
            TIM(161)=0.0
            TIM(162)=20.0
            X1(161)=0.0
            X1(162)=2.0
            X2(161)=0.0
            X2(162)=2.0
            UA1(161)=0.0
            UA1(162)=0.5
            UA2(161)=0.0
            UA2(162)=1.0
            DO 30 K=1,80
         30 READ(5,5000) TDO(K),TBOD(K)
            DO 40 K=1,80
            DO(2*K-1)=TDO(K)
            DO(2*K)=TDO(K)+(TDO(K+1)-TDO(K))/2
            BOD(2*K-1)=TBOD(K)
         40 BOD(2*K)=TBOD(K)+(TBOD(K+1)-TBOD(K))/2
       5000 FORMAT(2F4.1)
            TIM(1)=0.0
            B1=-3.44
            B2=2.91
            B3=0.0169
            B4=-0.1910
            WRITE(6,200) B1,B2,B3,B4
            X1(1)=6
            U2=0
            X3=0
            X4=0
            DO 50 K=1,159
            TIM(K+1)=TIM(K)+0.5
            XX1=(0.9417+U2)
            XX2=(1.0617+U2)
            A=EXP(-XX1*T)
            C=EXP(-XX2*T)
            B=-2.67*(A-C)
            D=(0.56/XX1)*(1-A)
            E=-(.18/(XX1*XX2))*(1-XX2*A/.12+XX1*C/.12)
            F=.56*(1-C)/XX2
            G=1.57*(1-A)/XX1-.32*3.71/(XX1*XX2)*(1-XX2*A/.12+XX1*C/.12)
            H=3.71*(1-C)/XX2
            AA=(1-A)/XX1
            BB=2*(1-A)/XX1-6.4/(XX1*XX2)*(1-XX2*A/.12+XX1*C/.12)
            CC=20*(1-C)/XX2
            IF(K.GT.1) GO TO 45
            X2(1)=(F*6.0+H+CC*U2)/(1-C)
            U1=(X1(1)*(1-A)-B*X2(1)-D*10.0-E*6.0-G-BB*U2)/AA
            UA1(1)=U1
            UA2(1)=.19
      C AERATION CONTROL INPUT
         45 UA1(K+1)=UA1(K)
            ER=X1(K)-X1(1)
            IF(ER.GT.0) GO TO 78
         49 X3=X3-ER
            UA1(K)=B1*ER+B2*X3+U1
            IF(UA1(K).LT.0) UA1(K)=0
            GO TO 250
         78 IF(UA1(K).EQ.0) GO TO 250
            GO TO 49
      C     EFFLUENT DISCHARGE CONTROL
        250 ER1=X1(K)-X1(1)
            UA2(K+1)=UA2(K)
            IF(ER1.LT.0) GO TO 202
        201 X4=X4-ER1
            U2=B3*ER1+B4*X4
            IF(U2.LT.-.19) U2=-.19
            UA2(K)=(0.19+U2)*15.1
            GO TO 350
        202 IF(UA2(K).EQ.0) GO TO 350
            GO TO 201
        350 X2(K+1)=C*X2(K)+F*BOD(K)+H+CC*U2
         50 X1(K+1)=A*X1(K)+B*X2(K)+D*DO(K)+E*BOD(K)+G+AA*UA1(K)+BB*U2
            UA1(K+1)=UA1(K)
            UA2(K+1)=UA2(K)
            CALL XPLOT(TIM,X1,X2,UA1,UA2,1)
            WRITE(6,6010)
       6010 FORMAT('OUTPUT DO MEASURED TWICE DAILY')
            WRITE(6,200)X1
        200 FORMAT(10(2X,F8.2))
            WRITE(6,6011)
       6011 FORMAT('OUTPUT BOD MEASURED TWICE DAILY')
            WRITE(6,200) X2
            WRITE(6,6012)
       6012 FORMAT(' INPUT AERATION CONTROL')
            WRITE(6,200) UA1
            WRITE(6,6014)
       6014 FORMAT('EFFLUENT DISCHARGE CONTROL')
            WRITE(6,200) UA2
            STOP
            END
#END OF FILE
#
```

```
 LIST SEW1
>      C PROGRAM TO SIMULATE CLOSED LOOP RIVER SYSTEM (1-CONTROL, TEMPERATURE
>      C DEPENDENT PARAMETERS)
>            REAL X2(322), X1(322),XDO(160),XBOD(160),UA1(322),UA2(322),TDO(80)
>            REAL TBOD(80),TEMP(322),TIM(322),BOD(320),DO(320)
>            T=0.25
>            TIM(322)=15.01
>            X1(321)=0.0
>            X1(322)=2.0
>            X2(321)=0.0
>            X2(322)=2.0
>            UA1(321)=0.0
>            UA1(322)=0.5
>            UA2(321)=0.0
>            UA2(322)=1.0
>            TEMP(321)=0.0
>            TEMP(322)=2.5
>            TIM(1)=0.0
>            DO 30 K=1,80
>         30 READ(5,5000) TDO(K),TBOD(K)
>            DO 40 K=1,80
>            XDO(2*K-1)=TDO(K)
>            XDO(2*K)=TDO(K)+(TDO(K+1)-TDO(K))/2
>            XBOD(2*K-1)=TBOD(K)
>         40 XBOD(2*K)=TBOD(K)+(TBOD(K+1)-TBOD(K))/2
>            DO 44 K=1,160
>            DO(2*K-1)=XDO(K)
>            DO(2*K)=XDO(K)+(XDO(K+1)-XDO(K))/2
>            BOD(2*K-1)=XBOD(K)
>         44 BOD(2*K)=XBOD(K)+(XBOD(K+1)-XBOD(K))/2
>       5000 FORMAT(2F4.1)
>            B1=-2.5221
>            B2=1.9514
>            B3=0.0
>            B4=0.0
>            DR=6.0
>            U2=0
>            X3=0
>            X4=0
>            X5=0
>            READ(7,560) XJ,X1(1),U1
>        560 FORMAT(3F5.1)
>            DO 50 K=1,319
>            TIM(K+1)=TIM(K)+0.25
>            XT=-4.30*SIN(0.0172*(XJ)-0.40)+9.19
>            TEMP(K)=XT
>            A1=0.1444*EXP(0.025*XT)
>            A2=0.1748*EXP(0.0464*XT)
>            CS=14.462*EXP(-0.0210*XT)
>            DB=0.0590*(25.0-0.028*(XT-30)*(XT-30))
>            XX1=A1+0.7417+U2
>            XX2=A2+0.7417+U2
>            CD=XX2-XX1
>            A=EXP(-XX1*T)
>            C=EXP(-XX2*T)
>            B=-(A2/(A2-A1))*(A-C)
>            D=(0.56/XX1)*(1-A)
>            E=-0.56*A2*(1-(XX2/(XX2-XX1))*A+(XX1/(XX2-XX1))*C)
>            F=(0.56/XX2)*(1-C)
>            G=((A1*CS-DB+0.37)/XX1)*(1-A)-A2*3.71*(1-(XX2/CD)*A+(XX1/CD)*C)
>            H=3.71*(1-C)/XX2
>            AA=(1-A)/XX1
>            BB=2*(1-A)/XX1-6.4/(XX1*XX2)*(1-XX2*A/.12+XX1*C/.12)
>            CC=20*(1-C)/XX2
>            IF(K.GT.1) GO TO 45
>            X2(1)=(F*6.0+H+CC*U2)/(1-C)
>            IF(U1.LT.0.0) U1=0.0
>            UA1(1)=U1
>      C AERATION CONTROL INPUT
>         45 ER=X1(K)-DR
>            IF(ER.GT.0) GO TO 78
>         49 X3=X3-ER
>            UA1(K)=B1*ER+B2*X3+U1
>            IF(UA1(K).LT.0) UA1(K)=0
>            GO TO 350
>         78 IF(K.GT.1) GO TO 88
>            IF(UA1(K).EQ.0) GO TO 350
>            GO TO 49
>         88 IF(UA1(K-1).EQ.0) GO TO 351
>            GO TO 49
>        351 UA1(K)=UA1(K-1)
>        350 X2(K+1)=C*X2(K)+F*BOD(K)+H+CC*U2
>            X1(K+1)=A*X1(K)+B*X2(K)+D*DO(K)+E*BOD(K)+G+AA*UA1(K)+BB*U2
>            WRITE(6,200) X1(K),X2(K),UA1(K),X3,XT
>        200 FORMAT(10(2X,F8.2))
>         50 XJ=XJ+0.25
>            WRITE(6,200) XJ
>            STOP
>            END
#END OF FILE
#
```

```
LIST SEW
C PROGRAM TO SIMULATE CLOSED LOOP RIVER SYSTEM (2-CONTROL, TEMPERATURE
C DEPENDENT PARAMETERS)
      REAL X2(322), X1(322),XDO(160),XBOD(160),UA1(322),UA2(322),TDO(80)
      REAL TBOD(80),TEMP(322),TIM(322),BOD(320),DO(320)
      T=0.25
      TIM(322)=15.01
      X1(321)=0.0
      X1(322)=2.0
      X2(321)=0.0
      X2(322)=2.0
      UA1(321)=0.0
      UA1(322)=0.5
      UA2(321)=0.0
      UA2(322)=1.0
      TEMP(321)=0.0
      TEMP(322)=2.5
      TIM(1)=0.0
      DO 30 K=1,80
   30 READ(5,5000) TDO(K),TBOD(K)
      DO 40 K=1,80
      XDO(2*K-1)=TDO(K)
      XDO(2*K)=TDO(K)+(TDO(K+1)-TDO(K))/2
      XBOD(2*K-1)=TBOD(K)
   40 XBOD(2*K)=TBOD(K)+(TBOD(K+1)-TBOD(K))/2
      DO 44 K=1,160
      DO(2*K-1)=XDO(K)
      DO(2*K)=XDO(K)+(XDO(K+1)-XDO(K))/2
      BOD(2*K-1)=XBOD(K)
   44 BOD(2*K)=XBOD(K)+(XBOD(K+1)-XBOD(K))/2
 5000 FORMAT(2F4.1)
      B1=-3.44
      B2=2.91
      B3=0.0169
      B4=-0.1910
      DR=6.0
      U2=0
      X3=0
      X4=0
      X5=0
      READ(7,560) XJ,X1(1),U1
  560 FORMAT(3F5.1)
      DO 50 K=1,319
      TIM(K+1)=TIM(K)+0.25
      XT=-4.30*SIN(0.0172*(XJ)-0.40)+9.19
      TEMP(K)=XT
      A1=0.1444*EXP(0.025*XT)
      A2=0.1748*EXP(0.0464*XT)
      CS=14.462*EXP(-0.0210*XT)
      DB=0.0590*(25.0-0.028*(XT-30)*(XT-30))
      XX1=A1+0.7417+U2
      XX2=A2+0.7417+U2
      CD=XX2-XX1
      A=EXP(-XX1*T)
      C=EXP(-XX2*T)
      B=-(A2/(A2-A1))*(A-C)
      D=(0.56/XX1)*(1-A)
      E=-0.56*A2*(1-(XX2/(XX2-XX1))*A+(XX1/(XX2-XX1))*C)
      F=(0.56/XX2)*(1-C)
      G=((A1*CS-DB+0.37)/XX1)*(1-A)-A2*3.71*(1-(XX2/CD)*A+(XX1/CD)*C)
      H=3.71*(1-C)/XX2
      AA=(1-A)/XX1
      BB=2*(1-A)/XX1-6.4/(XX1*XX2)*(1-XX2*A/.12+XX1*C/.12)
      CC=20*(1-C)/XX2
```

```
      IF(K.GT.1) GO TO 45
      X2(1)=(F*6.0+H+CC*U2)/(1-C)
      IF(U1.LT.0.0) U1=0.0
      UA1(1)=U1
      UA2(1)=.19
C AERATION CONTROL INPUT
   45 ER=X1(K)-DR
      UA1(K+1)=UA1(K)
      IF(ER.GT.0) GO TO 78
   49 X3=X3-ER
      UA1(K)=B1*ER+B2*X3+U1
      IF(UA1(K).LT.0) UA1(K)=0
      GO TO 250
   78 IF(K.GT.1) GO TO 88
      IF(UA1(K).EQ.0) GO TO 250
      GO TO 49
   88 IF(UA1(K-1).EQ.0) GO TO 250
      GO TO 49
C     EFFLUENT DISCHARGE CONTROL
  250 ER1=X1(K)-DR
      UA2(K+1)=UA2(K)
      IF(ER1.LT.0) GO TO 202
  201 X4=X4-ER1
      U2=B3*ER1+B4*X4
      IF(U2.LT.-.19) U2=-.19
      UA2(K)=(0.19+U2)*15.1
      GO TO 350
  202 IF(K.GT.1) GO TO 99
      IF(UA2(K).EQ.0) GO TO 350
      GO TO 201
   99 IF(UA2(K-1).EQ.0) GO TO 350
      GO TO 201
  350 X2(K+1)=C*X2(K)+F*BOD(K)+H+CC*U2
      X1(K+1)=A*X1(K)+B*X2(K)+D*DO(K)+E*BOD(K)+G+AA*UA1(K)+BB*U2
      WRITE(6,200) X1(K),X2(K),UA1(K),UA2(K),XT
  200 FORMAT(10(2X,F8.2))
   50 XJ=XJ+0.25
      WRITE(6,200) XJ
      STOP
      END
#END OF FILE
#
```

```
 LIST WPT
>      1      LABEL WATER POLLUTION CONTROL- TEMPERATURE EFFECT
>      1.2    LABEL SIMULATION OF CONTINUOUS SYSTEM (1-CONTROL)
>      2      PARAMETER X=1.0, IC1=6.0, UAB=0.31
>      3      INITIAL
>      3.1    TEMP=-4.30*SIN(0.0172*(X+TIME)-0.40)+9.19
>      3.2    A1=0.1444*EXP(0.025*TEMP)
>      3.3    A2=0.1748*EXP(0.0464*TEMP)
>      3.4    CS=14.462*EXP(-0.0210*TEMP)
>      3.5    DB=0.0590*(25.0-0.028*(TEMP-30)*(TEMP-30))
>      5      IC2=(0.556*6.0+3.7)/(A2+0.74)
>      6      IC3=0.0
>      8      DYNAMICS
>     10      TEMP=-4.30*SIN(0.0172*(X+TIME)-0.40)+9.19
>     10.8    A1=0.1444*EXP(0.025*TEMP)
>     11.6    A2=0.1748*EXP(0.0464*TEMP)
>     12.4    CS=14.462*EXP(-0.0210*TEMP)
>     13.2    DB=0.0590*(25.0-0.028*(TEMP-30)*(TEMP-30))
>     14      BOD=3.*STEP(0.)+.4*RAMP(4.)-.4*RAMP(14.)+.25*RAMP(24.)...
>     15       -1.25*RAMP(28.)+1.075*RAMP(31.)-.075*RAMP(51.)+...
>     16       2.5*RAMP(53.)-5.*RAMP(55.)+2.5*RAMP(57.)-.2*RAMP(60.)...
>     17       +.5*RAMP(70.)
>     18      DO=10.*STEP(0.)+RAMP(10.)-RAMP(10.5)-2.*RAMP(21.)+...
>     19        2.*RAMP(21.5)+RAMP(31.)-RAMP(31.5)-3.*RAMP(41.)+...
>     20       3.*RAMP(41.5)+RAMP(51.)-RAMP(51.5)
>     20.1    ERR=X1-6.0
>     20.2    X3=INTGRL(IC3,-ERR)
>     20.3    X3B=-3.26*ERR
>     20.4    UA1=8.82*X3+X3B+UAB
>     20.42   UA=LIMIT(0.0,1000.0,UA1)
>     20.5    X1D=-(A1+0.74)*X1-A2*X2+0.556*DO+A1*CS-DB+.37+UA
>     21      X2D=-(A2+0.74)*X2+0.556*BOD+3.7
>     22      X1=INTGRL(IC1,X1D)
>     23      X2=INTGRL(IC2,X2D)
>     24      TIMER DELT=0.1, FINTIM=80,PRDEL=0.5
>     25      PRINT X1,X2,TEMP,UA
>     26      END
>     26.1    PARAMETER X=93.0, IC1=6.61, UAB=0.0
>     26.2    END
>     26.3    PARAMETER X=183.0, IC1=6.20, UAB=0.0
>     26.4    END
>     26.5    PARAMETER X=274.0, IC1=6.0, UAB=0.70
>     26.6    END
>     27      STOP
>     28      ENDJOB
#END OF FILE
#
```

```
LIST WFD
>   LABEL WATER POLLUTION CONTROL- TEMPERATURE EFFECT
>   LABEL SIMULATION OF CONTINUOUS SYSTEM (2-CONTROL)
>   PARAMETER X=1.0, IC1=6.0, UAB=0.31
>   INITIAL
>   TEMP=-4.30*SIN(0.0172*(X+TIME)-0.40)+9.19
>   A1=0.1444*EXP(0.025*TEMP)
>   A2=0.1748*EXP(0.0464*TEMP)
>   CS=14.462*EXP(-0.0210*TEMP)
>   DB=0.0590*(25.0-0.028*(TEMP-30)*(TEMP-30))
>   IC2=(0.556*6.0+3.7)/(A2+0.74)
>   IC3=0.0
>   UEB=2.8
>   VM=15.1
>   DYNAMICS
>   TEMP=-4.30*SIN(0.0172*(X+TIME)-0.40)+9.19
>   A1=0.1444*EXP(0.025*TEMP)
>   A2=0.1748*EXP(0.0464*TEMP)
>   CS=14.462*EXP(-0.0210*TEMP)
>   DB=0.0590*(25.0-0.028*(TEMP-30)*(TEMP-30))
>   BOD=3.*STEP(0.)+.4*RAMP(4.)-.4*RAMP(14.)+.25*RAMP(24.)...
>    -1.25*RAMP(28.)+1.075*RAMP(31.)-.075*RAMP(51.)+...
>    2.5*RAMP(53.)-5.*RAMP(55.)+2.5*RAMP(57.)-.2*RAMP(60.)...
>    +.5*RAMP(70.)
>   DO=10.*STEP(0.)+RAMP(10.)-RAMP(10.5)-2.*RAMP(21.)+...
>     2.*RAMP(21.5)+RAMP(31.)-RAMP(31.5)-3.*RAMP(41.)+...
>    3.*RAMP(41.5)+RAMP(51.)-RAMP(51.5)
>   ERR=X1-6.0
>   X3=INTGRL(IC3,-ERR)
>   UA1=-2.73*X3-1.59*ERR+UAB
>   UA=LIMIT(0.0,1000.0,UA1)
>   UE1=13.66*ERR-52.11*X3+UEB
>   UE=LIMIT(0.0,1.0E12,UE1)
>   X1D=-(A1+0.556+UE/VM)*X1-A2*X2+0.556*DO+A1*CS-DB+2*UE/VM+UA
>   X2D=-(A2+0.556+UE/VM)*X2+0.556*BOD+20*UE/VM
>   X1=INTGRL(IC1,X1D)
>   X2=INTGRL(IC2,X2D)
>   TIMER DELT=0.1, FINTIM=80,OUTDEL=1.0, PRDEL=.5
>   PRINT X1,X2,UA,UE,X3,TEMP
>   END
>   PARAMETER X=93.0, IC1=6.61, UAB=0.0
>   END
>   PARAMETER X=183.0, IC1=6.20, UAB=0.0
>   END
>   PARAMETER X=274.0, IC1=6.0, UAB=0.70
>   END
>   STOP
>   ENDJOB
#END OF FILE
#
```

B30185